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Aspects of Structural Reliability

In Honor of R. Rackwitz

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Design bases vs. expected performance for long span suspension bridges

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ABSTRACT: In a sort of dialog between a designer and an expert in structural reliability, the theme is to provide the bases of design for a long-span suspension bridge. Nevertheless, the discussion applies to many large civil engineering structural systems. Five main aspects are emphasized: action definition, sub-structuring, interactions, robustness and monitoring.

1 INTRODUCTION

Speculating by dialogs was an attitude at the age of the ancient Greek philosophers, as well as it was nearly the rule at the embryo of modern Mechanics (Galileo, 1630). Today, that style of arguing in scientific communications is lost. But the reader can regard this paper as the product of dialogs between two characters. The first is engaged in the formulation of modern bases for the design of a long-span suspension bridge. The second character serves as the knowledge basis of the most recent developments in structural reliability theory and probabilistic risk assessment.

The pioneering paper by Freudenthal (1956) in this area is dated fifty years ago, but only twenty years later the solution of the inherent computational problems was successful approached (Rackwitz & Fiessler, 1978; Olivi, 1980). In the Eighties a bundle of books were published on the topic (Ditlevsen, 1981; Ang & Tang, 1984; Augusti et al., 1984; Madsen et al., 1986; Melchers, 1987; Casciati & Faravelli, 1991; Ditlevsen & Madsen, 1996; Casciati & Roberts, 1996) and the Nineties generated the concept of "performance based design" (Fujitani et al., 2005) and the associated probabilistic model code format (JCSS, 2001).

Within this framework one is expecting that only minor obstacles are is met when the bases for the design of a long span suspension bridge are demanded (Gimsing, 1997; Calzona, 2005). The design bases of bridges of recent realization (Wong, 2003; Kitagawa 2003) are considered as a starting point. But the way is not straightforward. Five aspects will be discussed in detail:

1) the definition of time variant actions;
2) the ability to sub-structure the complex structural problem;
3) the interactions of the structural system with the foundation and the environment;
4) the way to manage accidental actions and related robustness issues;
5) the interconnection between structural reliability and structural monitoring.

This paper does not provide definite answers. It just underlines the problems and shows feasible solutions, if any. But the impression is that several points still require a better formulation of the problem, a more rigorous treatment and a final systematization of the pertinent theoretical background.

2 THE STRUCTURAL LIFETIME

The Brooklyn suspension bridge celebrated its centenary and it is likely it will celebrate several additional decades. No matter, therefore, that in designing the Stonecutters (HKHD, 2001) cable-stayed bridge in Hong Kong, the responsible authority was demanding an economic lifetime L of 120 years. Assume now that the authority X be involved in the design of a new suspension bridge and its span be one time and half longer than the presently longest suspension bridge in the world, the Akashi Kaikyo bridge (Kitagawa, 2003). It is likely that the authority X will ask for an economical lifetime of L = 200 years!

Immediate consequences of the selection of such a number are found both in the definition of the actions and in the modeling of the resistance.

2.1 *Defining the time variant actions*

The Stonecutters authority introduced the action “earthquake” through design spectra of assigned return periods, but different for the serviceability limit states and the ultimate limit states. The latter value was 2400 years, or simply the upper-fractile 5% of the distribution of the maximum in 120 years. It is the application of standard structural reliability features. When the authority X applies the same reasoning to its bridge with L = 200 years, a return period of 4000 years is found.

Moreover, in view of that later will be referred to as robustness, the authority X also needs to define action levels more devastating than those associated with the ultimate limit states in view of checking the “structural integrity” after catastrophic events. Extending the above reasoning, one fixes the 2% upper-fractile, thus achieving a return period of 10000 years (it was 6000 years for the Stonecutters case). Such a way, could be easily extended to the definition of other time variant actions as wind, snow, temperature and so on.

A structural engineers will not be disturbed by these digits. The collection of wind time histories started as a need for the aeronautic traffic and their records date back to the Fifties. The collection of accelerometric data comes after the introduction of the measurement instrument (Housner, 1947) and existing databases date back to the Sixties or Seventies. The structural engineer, therefore, does not rely on statistics, but rather on theoretical models statistically built on the available data.

But, consulting an expert in hydrology, whose data date back to the 19th century, the following two main objections arise:

1) on the basis of the available statistics, one can express confident estimates on return periods not longer than 300 years;
2) the basic assumption is that nature repeats its behavior without modifications; but it is well known that the period of nature cycles is just of some centuries.

Terminology must therefore be corrected in order to avoid misunderstandings. Since the realization of a large infrastructure cannot be preceded by an extremely long period of data collection, the available statistics are converted into probabilistic models on the basis of which the design values of the time variant actions are selected. These values can be derived as the ones with return periods of millenniums, but in this case they have to be regarded as conventional values and their specification in terms of return period should be completed by an attribute (f.i., conventional) clarifying their nature.

Unfortunately, a simple rule which sees the conventional return period values, to be used for the different limit states, as generated by the desired extension of the lifetime does not find unanimous consensus. A wind engineer, in fact, is anchored to the definition of the reference wind velocity with explicit return period of 50 years (JCSC, 2001; Carassale & Solari, 2005). This value can be increased, when necessary, to the lifetime period (HKHD, 1990). From this reference value, then, higher design values are achieved by introducing factors higher than 1, avoiding their interpretation in terms of return period. A consistent general definition of all time variant actions would certainly help, mainly because safety factors of uncertain source could be definitively replaced by statistics when available. On the other side, compromises between different scientific communities (as the ones of wind and earthquake engineering) do not result straightforward.

One should also distinguish between statistics collected in situ and information derived from a database. A long span bridge connecting two sea coasts, when realized, will see its deck at 65 m on the sea level, in order to allow the naval traffic below it. Moreover, the mid-span will be more than 1 km away from each of the two coasts. There is no way to measure the wind speed at the deck mid-span before the construction would have been completed. Numerical models are used to reconstruct the wind phenomenon at that point and along the deck in general, but their accuracy could be afflicted by some forgotten (or simply unexpected) local phenomena. In particular the existence of privileged wind directions could seriously affect the final design specifications. An adequate design, therefore, should ask for measurement during constructions (which often takes several years) and mainly during the first decade of the bridge utilization, in order to update the load models and, hence, the estimated reliability. A scenario including the bridge retrofitting should also be listed.

A further aspect must be emphasized: natural phenomena are rarely linear. It is difficult that a natural action can simply be scaled by its intensity to simulate less and less frequent events. In the seismic event case, given a site, the first data elaboration aims at establishing a link between peak ground acceleration (PGA) and return period: the model of seismo-genetic zones hold up to return periods of 1000 or 2000 years, and then by mere extrapolation one moves to longer periods. But the goal is the definition of time histories of the seismic action in terms of velocity, displacement and acceleration; they will serve as input to the dynamic transient analyses the design requires. For a suspension bridge, the motion intensity is not the only parameter of interest. One must watch relative difference of the motion at the towers, and between these and the anchorages, as well as differential amplifications due to the different mechanical properties of the foundation rocks and the motion (horizontal vs. vertical) incidence. The situation of Figure 1 cannot be a priori excluded. To impose simultaneous time histories at the four supporting areas is unacceptable.

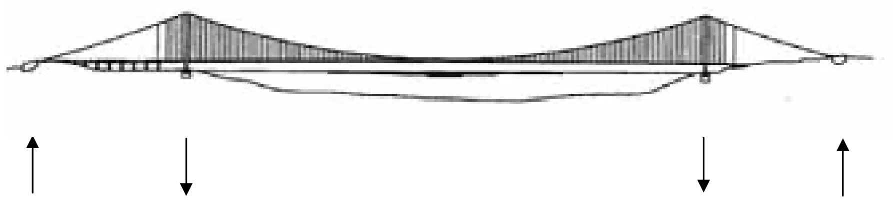

Figure 1 – Possible negative seismic excitation scenario.

The seismic motion must therefore be re-constructed starting from the actual knowledge on geology, morphology and seismology at the site. The characteristics and the magnitude of the event depend on the properties of the seismic source, on the seismic waves pattern between source and site and on the features of the more superficial soil layers These three aspects are the basis of the simulation of design time histories. Historical catalogs and instrumental records support the identification of source zones and associated magnitude of the strongest expected events in the area. For each of these areas the relevant return period is estimated. The signals of the three acceleration components are than propagated from the source to the four bridge support areas. It is worth noting that such an approach is respectful of the physical properties of the seismic events:

a. the frequency content depends on the magnitude and hence, for the same source, on the return period;
b. the motion intensity varies in the four support areas depending on the provenience azimuth;
c. the provenience direction establishes the delay at the four support areas;
d. geometry and nature of the most superficial soil layers alter the intensity of the signals and the relative delays.

2.2 *Defining the resistance features*

The Brooklyn bridge suspension cables are giving its supervising authority a sound expertise on their durability (Yanev, 2005). But they were produced with the technology of more than one century ago! This expertise cannot be exported to different bridges. The enemy from which one has to defend is the corrosion, and still reliable models of its progress are lacking. The Akashi designers adopted an advanced de-humidification process to extend the cable life and a special painting (coating) to protect the steel structural components. Estimates of the reliability one can achieve by adopting these technologies are presently lacking.

In such a protective frame, assuming deteriorating parts and devices are replaced periodically, an extended lifetime could only find obstacle in the resistance to fatigue. Usual loading spectra are helpless since the 10 million of cycles will be likely achieved within a 200 years lifetime. The strength of the material undergoing so many cycles must be limited to its asymptotic lower bound detected by fatigue tests.

3 SUB-STRUCTURING

A suspension bridge is a structural systems supported in four different areas: the foundations of the two towers and the two cable anchorages. An accurate numerical models would see (Figure 2) four foundation soil volumes, two pylons, two or more cables running from anchorage to anchorage and supported on the top of the pylons, several hangers supporting the deck transversal beams on which the longitudinal beams insist. The structural analysis software runs on standard personal computers, but every transient dynamic analysis could required days of elaboration. This macroscopic model comes together with mesoscopic and microscopic model, which allow the designer the study of the details and their specification (Figure 2).

As observed in (Faravelli & Bigi, 1990), a structural analysis under uncertainty cannot rely on usual simplification assumptions as those based on symmetry: the geometrical symmetry is easily realized, but the symmetry of the realization of a random field is certainly a strong questionable assumption. In (Faravelli, 1989; Breitung & Faravelli, 1996), it was theorized that a response model depending on N variables can be expressed in a reduced space of size n by the form:

$$g(\boldsymbol{x}, \boldsymbol{y}) = F(\boldsymbol{x}) + \varepsilon(\boldsymbol{x}, \boldsymbol{y}) \qquad (1)$$

where the whole model is defined in the space of size N of the variables $\boldsymbol{x}$ and $\boldsymbol{y}$, the model $F(\,)$ in the n size space of the variables $\boldsymbol{x}$ and $\varepsilon(\boldsymbol{x}, \boldsymbol{y})$ takes into account both the pure error and the effect of the space reduction. The approach, however, is successful when the variability in the error term can be quantized in a few units per cent.

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Assessment of existing structures – On the applicability of the JCSS recommendations

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ABSTRACT: The Joint Committee on Structural Safety (JCSS) document on existing structures which is published under RILEM provides general guidelines on assessment of existing structures, methodologies for reliability updating, acceptability and target safety criteria. General rules are supplemented by examples and case studies. The scope of the document and the benefits related to its applicability are outlined in this contribution.

1 INTRODUCTION

In 1971 the Liaison Committee which co-ordinates the activities of six international associations in Civil Engineering FIB, CIB, ECCS, IABSE, IASS and RILEM, created a Joint Committee on Structural Safety (JCSS), with the aim of improving the general knowledge in structural safety. After reorganization in 1992 the JCSS set as a long term goal the development of a probabilistic model code for new and for existing structures. In 2001 the JCSS (2001)has published under RILEM a general document on probabilistic assessment of existing structures.

This contribution gives a short introduction to the JCSS document on existing structures and discusses its applicability on the basis of actual case studies. Updating procedures based on new data are applied. Special emphasis is paid to risk acceptance criteria and associated target safety values. Conclusions for future developments are provided.

2 JCSS RECOMMENDATIONS

The need to assess the reliability of an existing structure may arise from a number of causes such as deviations from the original project description, adverse results of a periodic investigation of its state, etc. A typical actual example is the reassessment of roofs under the extreme snow load. During the reassessment procedures typical questions which need to be answered are:

- What type of inspections are necessary?
- What analyses shall be performed?
- What are the risks involved in further using the structure?
- What are the risk acceptance criteria to be considered?
- What type of measures shall be taken?

Such answers cannot be given by using a classical code approach. In addition one key point is that new information becomes available related to the state of the existing structure. Therefore there is an increasing need and consequently an increasing tendency to use probabilistic methods in the assessment of existing structures. The scope of the JCSS (2001) document is to provide such reliability based procedures and to illustrate them in characteristic examples and case studies.

The JCSS document (2001) is of educational type and provides reliability methods to be used in the structural reassessment. Tutorial examples and practical case studies are included as shown in the contents, which are as follows:

The document provides relevant information on how to process specific information about an existing structure, how to update its reliability based on such information, how to base decisions regarding maintenance, strengthening, upgrading etc. It is generally applicable for various materials and various structure types. The chapters of the JCSS document and the associated guidelines are summarized in the following paragraphs.

3 RELIABILITY UPDATING

Assessment of existing structures using methods of modern reliability theory should be seen as a successive process of model building, consequence evaluation and model updating by introduction of new information, by modification of the structure or by changing the use of the structure. The principle may be illustrated schematically as shown in Figure 1.

The analyses to be performed involve various steps:

- Formulation of prior uncertainty models
- Formulation of limit state functions
- Establishing posterior probabilistic models
- Performing prior, posterior and pre-posterior decision analysis
- Setting acceptable levels for the probability of failure.

The two first steps are briefly addressed together in order to introduce the philosophy of Bayesian probabilistic modelling in the assessment of existing structures. The next two points, however, are essential for the understanding of the framework of structural reassessment and are described in detail. The respective methodological aspects are provided and applied in an educational example. The issue of setting acceptable failure probabilities is central both for reliability based design and reliability based assessment of structures. This issue is considered in part 3 of the JCSS document.

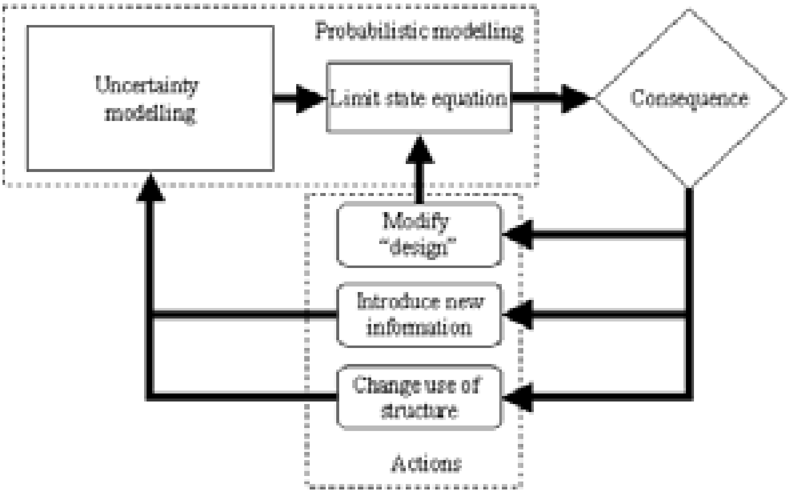

Figure 1. Bayesian probabilistic assessment of structures

4 ACCEPTABILITY AND TARGET CRITERIA

For reliability-based design and reliability assessment of existing structures acceptability limits or targets have to be set. Both quantities are not necessarily the same as they may result from partially different criteria. Also, they are not necessarily the same for structures to be designed and structures which already exist because the decision point (point in time where a decision is made whether some requirements are fulfilled or not) and thus the degree of information, the relative effort to control reliability and potential failure consequences is changed. Acceptability limits or targets may also differ depending on whether one considers an entire building facility including other than structural failure modes or the structure itself in the narrow sense. It is further necessary to distinguish between limits or targets set for facilities including human error in its various forms (design error, failure of quality management, operation failure, ignorance, etc.) and limits or targets where such failure causes are not included.

It should be considered also whether limits or targets are related to individual failure modes or the failure modes of a system and, in accordance with present practice, in relation to the failure consequences. Such failure consequences may include direct financial losses due damage and for demolition and reconstruction, injuries or even loss of human lives but also so-called intangibles like loss of future opportunities (for example, loss of public welfare, professional reputation, and the like). Limits or targets may be different depending on whose behalf (for example, user, builder and public) decisions are to be made.

Finally, in a probabilistic context, such limits or targets are not independent of the set of probabilistic models used to verify them. This yet incomplete list of aspects when setting limits or targets indicates that the question of setting targets or limits is all but trivial. They are nevertheless mandatory to render probabilistic design and/or reliability assessment of existing structures operational.

Such limits or targets have been developed for structural components and systems in the narrow sense by not including non-structural failures modes and by not including failures due to human error or ignorance as a function of relative cost of safety measure and of degree of failure consequences.

Much debate has been thereby going on whether to include human lives into cost benefit analyses and whether it is at all admissible to perform cost benefit analysis when human lives or injuries are involved in case of structural failures. This requires introduction of a monetary equivalent to save human life and limb into the analysis. More recent studies on behalf of the public use so-called compound social indicators. Social indicators are statistics that reflect some aspect of the quality of life in a society or group of individual. More specifically, they aim to reflect broadly accepted goals that may carry labels such as national development, high expectancy of quality-adjusted life, the common good or the public interest. Any undertaking (project, program or regulation, adoption of new therapy, etc.) that affects the public by changing health or risk and expenditure will have an expected impact on a compound social indicator. A positive net impact of an undertaking on the accepted social indicator will lend to support the undertaking.

For example, the Life Quality Index (LQI) is intended as an indicator for "quality-adjusted life expectancy. It is a function of the real gross national product (GNP) per person and year and the life expectancy at birth. If applied to the fatality risk for structural failure in developed countries it can be shown that in the nineties of the 20th century expenditures for the safety a human life have approximately a value of US$ 100000 per year or about US$ 4000000 per average life time. By using the LQI it is possible to include human losses when deriving optimal target reliability indices (see for example Rackwitz, 2002)

The target values for the ultimate limit states related to failure of structural members are presented in Table 1 adopted from ISO 2394 (1998). The values correspond to individual structural elements and to one year reference period and reflect both the code calibration experience and the aforementioned cost-benefit considerations. These values shall be considered in reliability analyses in association with the stochastic models for the influencing variables as described in the probabilistic model code [2]. In case of structures with extreme failure consequences the target values shall be defined using risk-benefit studies. For existing structures the costs of achieving a greater reliability level are usually high compared to structures under design. For this reason the target level for existing structures usually should be lower. A reduction of the target β value by 0.5 is recommended.

Table 1: Tentative target reliability indices β (and associated target failure probabilities) related to one year reference period and ultimate limit states

Relative Cost of Safety Measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Large	β=3.1($p_F \approx 10^{-3}$)	β=3.3($p_F \approx 5x10^{-4}$)	β=3.7($p_F \approx 10^{-4}$)
Normal	β=3.7($p_F \approx 10^{-4}$)	β=4.2($p_F \approx 10^{-5}$)	β=4.4($p_F \approx 5x10^{-6}$)
Small	β=4.2($p_F \approx 10^{-5}$)	β=4.4($p_F \approx 5x10^{-5}$)	β=4.7($p_F \approx 10^{-6}$)

The grading for both the relative effort to achieve reliability and the expected failure consequences agrees also well with calculations provided in various studies.

It is further noted here that the relationship between the failure probability and the reliability index is expressed as:

$$\beta = - \Phi^{-1} (p_f) \tag{1}$$

where p_f is the probability of failure and $\Phi^{-1}(\cdot)$ is the inverse Gaussian distribution.

5 APPLICATIONS

5.1 *Reliability assessment of an existing reinforced structure*

An example of an existing reinforced concrete roof is indicated in Figure 2 and 3.

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A tribute to Rüdiger: On the Rackwitz-Fiessler algorithm for the FBC load combination model corrected by SORM

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This paper is a revised and extended version of a paper presented at ICOSSAR'05 in Rome, June 2005

ABSTRACT: The old stochastic load combination model of Ferry Borges and Castanheta and the corresponding extreme random load effect value is considered. The evaluation of the distribution function of the extreme value by use of a particular first order reliability method was first described in a celebrated paper by Rackwitz and Fiessler. The method has become known as the Rackwitz-Fiessler algorithm. To make the paper self-contained, the original RF algorithm is described herein allowing to introduce a modest but quite effective accuracy improving calculation following after termination of the RF calculations. The calculation gives a limit state curvature correction factor on the probability approximation obtained by the RF algorithm. This correction factor is based on Breitung's celebrated asymptotic formula. Example calculations with comparisons with exact results show an impressing accuracy improvement.

1 PROLOGUE

This small paper is meant as a tribute to the memory of professor Júlio Ferry Borges who was one of the early European pioneers in application of probability theory to evaluate structural safety and who was the first president of the more than 30 years old Joint Committee on Structural Safety (JCSS). Moreover the paper is a tribute to the TUM research group under the leadership of professor Rüdiger Rackwitz for essential contributions to FORM and asymptotic SORM as applied to limit state probability evaluation problems in the standard Gaussian space. The load model of the paper is included in the probabilistic model code of JCSS (Joint Committee on Structural Safety 2001), which has been developing through many years under the JCSS presidents Jörg Schneider, Rüdiger Rackwitz, and Ton Vrouwenvelder.

2 INTRODUCTION

Within the probabilistic reliability analysis the loads are in principle modeled as random variables that are functions of both time and position on the structure, that is, as random processes when they are considered as functions of time and random fields when their variation in space is in focus.

The load model considered in this paper is the FBC load model (after J. Ferry Borges and M. Castanheta, who suggested the model for code specification (Ferry Borges & Castanheta 1971)). The FBC load model is constructed in a particular simple way with respect to the time variation. The model is idealized to such a degree that one hardly can state that it reflects realistic details in the load history. On the other hand, the model captures the essential influence on the failure probability of random changes of the load level. Moreover, the structure of the model makes it well suited for load combination investigations in which additions are made of the effects of several load histories of this type. Another advantage is that the model is easy to describe in a code specification because it can be considered as a direct detailing of the idealized deterministic load configurations that are specified in most current load codes concerning statically acting loads.

Following (Ditlevsen & Madsen 2004) a scalar FBC process is defined as a sequence of rectangular load pulses of fixed duration τ following immediately after each other. The elements in the corresponding sequence of pulse amplitudes are mutually independent identically distributed random variables (that is, the sequence of amplitudes is a so-called Bernoulli sequence). The process starts at time $t = 0$. It is written as $X(t, \tau)$.

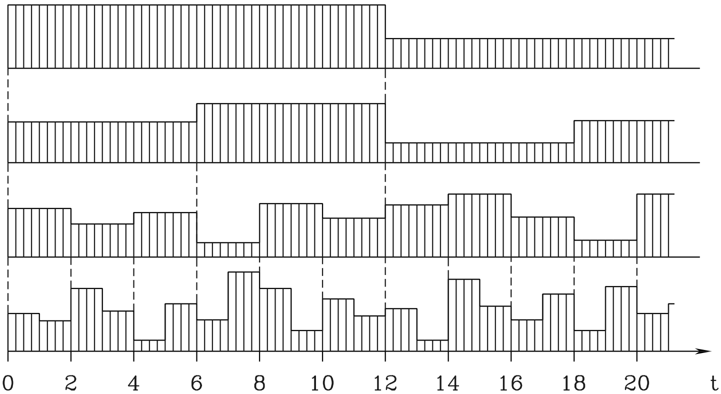

Figure 1. Realization of four-combination FBC process $[X_1(t, 12), X_2(t, 6), X_3(t, 2), X_4(t, 1)]$.

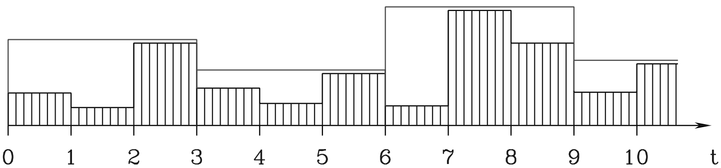

Figure 2. Realization of FBC process $X(t, 1)$ with corresponding realization of the T-duration envelope $X(t, 3)$ with $T = 3$.

An n-combination FBC process is a vector $[X_1(t, \tau_1), \ldots, X_n(t, \tau_n)]$ of scalar FBC processes ordered with respect to element number such that the pulse durations are never increasing, that is, such that $\tau_1 \geq \tau_2 \geq \ldots \geq \tau_n$, and which relative to each other have the particular property that τ_i / τ_j is an integer for all $i \leq j$, Fig. 1.

The T-duration envelope to an FBC process $X(t, \tau)$ is an FBC process denoted as $X(t, T)$, in which the pulse duration T is an integer multiple of τ and in which the amplitude is defined as the maximum of $X(t, \tau)$ over the considered pulse interval for $X(t, T)$, Fig. 2. For $T = \tau$ the envelope and the FBC process are identical.

Let the different FBC load processes contribute linearly to a load effect with positive influence coefficients. Moreover, assume that all load pulse amplitudes are non-negative with probability one. It is then sufficient to study the sum

$$X_1(t, \tau_1) + \ldots + X_n(t, \tau_n) \tag{1}$$

of the elements in an n-combination FBC process. We ask for the probability distribution of the maximal load effect within the time interval $[0, \tau_1]$. By considering Fig. 1 it is directly seen that the nth term $X_n(t, \tau_n)$ in the sum (1) can be replaced by the corresponding τ_{n-1}-envelope $X_n(t, \tau_{n-1})$ without changing the maximal load effect. Since

$$Z_{n-1}(t, \tau_{n-1}) \equiv X_{n-1}(t, \tau_{n-1}) + X_n(t, \tau_{n-1}) \tag{2}$$

is an FBC process, the n-combination problem is by this reduced to an $(n-1)$-combination problem corresponding to the $(n-1)$-combination FBC process $[X_1(t, \tau_1), \ldots, X_{n-2}(t, \tau_{n-2}), Z_{n-1}(t, \tau_{n-1})]$.

The distribution function of the amplitudes of $Z_{n-1}(t, \tau_{n-1})$ is the convolution

$$F_{Z_{n-1}(t,\tau_{n-1})} = \int_{-\infty}^{\infty} F_{X_n(t,\tau_{n-1})}(z - x) f_{X_{n-1}(t,\tau_{n-1})}(x)\, \mathrm{d}x \tag{3}$$

between the density function $f_{X_{n-1}(t,\tau_{n-1})}(x)$ and the distribution function

$$F_{X_n(t,\tau_{n-1})}(x) = [F_{X_n(t,\tau_n)}(x)]^{\tau_{n-1}/\tau_n} \tag{4}$$

for the τ_{n-1}-envelope of $X_n(t, \tau_n)$.

It follows from this reduction of the dimension of the problem that the distribution function of the maximal load effect is determined by $n - 1$ subsequent convolution integrations. Generally such a computation is difficult to do by use of standard analytical or numerical methods.

If it is assumed that all the amplitude distributions are absolutely continuous, and that only fractile values are of interest in the upper tail of the distribution of the maximal load effect, it is usually so that a sufficient accuracy for practical purposes can be obtained by first order reliability calculations (FORM).

Next section describes the particular application of the so-called normal tail approximation principle as it is tailored for the solution of the FBC n-combination problem under the assumption that all amplitude distributions are absolutely continuous. The iteration algorithm appeared for the first time in the often cited paper (Rackwitz & Fiessler 1978).

A true anecdote: I remember a reaction from Júlio Ferry Borges at an MIT workshop on structural load modeling hosted by Allin Cornell at a rural place in the vicinity of Boston in June 1976. When Rüdiger Rackwitz declared that he could evaluate the probability that the combination load exceeds a given level by just checking at a single point, Ferry Borges spontaneously and loudly bursted out “I don’t believe it”.

3 RACKWITZ-FIESSLER ALGORITHM FOR ABSOLUTELY CONTINUOUS DISTRIBUTION FUNCTIONS

Due to the recursive reduction of the n-combination problem it is sufficient first to consider the case $n = 2$. The RF algorithm computes an approximation to the value of both the distribution function and the density function of

$$Z_1(t, \tau_1) = X_1(t, \tau_1) + X_2(t, \tau_1) \tag{5}$$

for an arbitrary choice of the argument z in the following way. Choose a point (x_1, x_2) on the straight line

$$x_1 + x_2 = z \tag{6}$$

and determine (μ_1, σ_1) and (μ_2, σ_2) by use of $\Phi[(x_i - \mu_i)/\sigma_i] = F_i(x_i)$, $\varphi\left[(x_i - \mu_i)/\sigma_i\right]/\sigma_i = f_i(x_i)$, $i = 1, 2$, where $F_1 = F_{X_1(t,\tau_1)}$ and $F_2 = F_{X_2(t,\tau_1)}$ and where $\Phi(\cdot)$ and $\varphi(\cdot)$ are the standard normal distribution function and density function, respectively.

Hereby the density function and the distribution function of $Z_1(t, \tau_1)$ get their values at z approximated by the values at z of the density function and the distribution function, respectively, of the normal distribution with parameters

$$\mu = \mu_1 + \mu_2, \quad \sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{7}$$

It is obvious that the results depend on the approximation point (x_1, x_2). For each choice of z we therefore should look for the "best" approximation point. This is made by the "backward" part of the RF algorithm in the following way. A new approximation point (x_1, x_2) is chosen as the point on the straight line (6), at which the product of the two approximating normal density functions with parameters (μ_1, σ_1) and (μ_2, σ_2) has maximal value. Using the Lagrangian multiplier method this point is obtained by minimizing $[(x_1 - \mu_1)/\sigma_1]^2 + [(x_2 - \mu_2)/\sigma_2]^2 - 2\lambda(x_1 + x_2 - z)$. Setting the partial derivatives to zero we get $x_1 - \mu_1 = \lambda\sigma_1^2$ and $x_2 - \mu_2 = \lambda\sigma_2^2$, which by addition gives $z - \mu = \lambda\sigma^2$. Thus

$$(x_1, x_2) = (\mu_1 + \beta\alpha_1\sigma_1, \mu_2 + \beta\alpha_2\sigma_2) \tag{8}$$

in which

$$\beta = \frac{z - \mu}{\sigma}, \quad (\alpha_1, \alpha_2) = \left(\frac{\sigma_1}{\sigma}, \frac{\sigma_2}{\sigma}\right) \tag{9}$$

The procedure is repeated with start at the new approximation point (x_1, x_2). By continued iteration in this way a sequence of points $(x_{11}, x_{21}), (x_{12}, x_{22}), \ldots, (x_{1n}, x_{2n}), \ldots$ is generated. If the sequence converges to a point (x_1, x_2), this point is a locally most central point on the straight line (6) and

$$F_{Z_1}(z) \approx \Phi\left(\frac{z - \mu}{\sigma}\right), f_{Z_1}(z) \approx \frac{1}{\sigma}\varphi\left(\frac{z - \mu}{\sigma}\right) \tag{10}$$

For $n > 2$ the RF algorithm runs as follows. Choose an approximation point $(x_1, \ldots, x_n)$ on the hyperplane

$$x_1 + \ldots + x_n = z \tag{11}$$

and define the subsums

$$\begin{aligned} z_n &= x_n \\ z_{n-1} &= x_{n-1} + z_n \\ z_{n-2} &= x_{n-2} + z_{n-1} \\ &\vdots \\ z_2 &= x_2 + z_3 \\ z_1 &= x_1 + z_2 \end{aligned} \tag{12}$$

by which $z_1 = z$. In the first step of the algorithm, approximation values for the distribution function and the density function of $Z_{n-1}(t, \tau_{n-1})$ corresponding to the argument z_{n-1} are obtained as explained for $n = 2$ before entering the backward part of the algorithm. With these approximation values as input the same computation is made for

$$Z_{n-2}(t, \tau_{n-2}) = X_{n-2}(t, \tau_{n-2}) + Z_{n-1}(t, \tau_{n-2}) \tag{13}$$

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On temporal and spatial probabilistic engineering modeling

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ABSTRACT: The present paper addresses temporal and spatial modeling from the perspective of providing basis for societal decision making in regard to the further development, maintenance and safeguarding and societal infrastructure. Starting point is taken in the viewpoint that decisions concerning infrastructure projects should be optimized considering all potential benefits and losses which might arise during their entire life-cycle. For this purpose a systems representation in an intergenerational context is outlined which provides guidance on how risks may be consistently assessed considering benefits and losses caused by physical changes as well as perceived knowledge. The suggested systems representation is built up on three characteristics of the systems, namely; exposure, vulnerability and robustness. The detailed consideration of these in the modeling process as well as in the formulation of the decision problem provides aid in assessing direct and indirect consequences as well as on how these may efficiently be managed. The proposed modeling framework is illustrated and discussed taking basis in recent research on optimal engineering decision making considering maintenance planning of concrete structures, robustness assessment of structures and loss estimation for larger building stocks exposed to earthquake hazards.

1 INTRODUCTION

The services provided by the engineering community to society are of tremendous importance. Engineers in the past have created the societal infrastructure and thereby facilitated the societal development as we see it today. The societal infrastructure can be seen as the backbone of society without which there would be no civilization as we know it. Among significant engineering achievements count the numberless roadway bridges, tunnels, roadway systems, water, waste and energy distribution systems, structures for housing and industry as well as facilities for exploitation and distribution of various types of energy.

Until recently developments of society have been undertaken with only little or no concern in regard to the availability of the resources required for continued developments as well as the impact of societal activities on the qualities of our environment. The main focus has so far directed on the feasibility of various societal activities measured on the same scales as any other economical transaction in the free marked. Within the recent years the need for sustainable societal developments has become a general concern both at the political and the operational level in society, (Development 1987). Not least in the context of decision making in regard to the further development, maintenance and safeguarding of existing infrastructure decisions made at the present may have significant effect on the generations in the future.

Whenever decisions are made committing or using societal resources for development, maintenance and safeguarding infrastructure, society looses access to resources which in other ways might have been used to improve the life quality of the individuals of society. Therefore, societal decisions in this regard must be made such as to achieve an appropriate balance between investments and the benefit achieved through better performing infrastructure. This seemingly simple problem is, however, not so easy to frame in practice and comprises one of the main challenges in engineering decision making. How can we consistently assess the performance of societal infrastructure in such a way as to facilitate decisions which optimizes the benefit for society - now and in the future? The answer to this question is complicated due to the fact that the benefits are influenced by many uncertainties. Not only are there significant uncertainties and lack of knowledge associated with the modeling of the hazards to which the societal infrastructure is exposed but also the performance of the infrastructure for given hazard exposures is affected by uncertainties.

Typical engineering problems such as design, assessment, inspection and maintenance planning and decommissioning are consequently decision problems subject to a combination of natural inherent, modeling and statistical uncertainties. This fact was fully appreciated already some 30 – 40 years ago and since then the Bayesian decision theory together with Bayesian probabilistic modeling has formed the cornerstones of what is now commonly understood as modern methods of structural reliability see e.g. (Freudenthal 1947), (Turkstra, 1970) and (Ferry Borges and Castanheta, 1971). Within the recent years, taking basis in the theory of decision analysis see e.g. (Von Neumann and Morgenstern 1944; Raiffa and Schlaifer 1961), applied decision theory has been developed significantly for the support of a broad variety of engineering decisions, including structural design, assessment and maintenance of existing structures, planning of laboratory testing and on-site investigations as well as assessment and management of industrial and natural hazards.

In the present paper starting point is taken in the viewpoint that decisions concerning infrastructure projects should be optimized considering all potential benefits and losses which might arise during their entire life-cycle. For this purpose a systems representation in an intergenerational context is outlined which provides guidance on how risks may be consistently assessed considering benefits and losses caused by physical changes as well as perceived knowledge. The suggested systems representation is built up on three characteristics of the systems, namely; exposure, vulnerability and robustness. The detailed consideration of these in the modeling process as well as in the formulation of the decision problem provides aid in assessing direct and indirect consequences. The suggested modeling framework is illustrated and discussed taking basis in recent research on exposure modeling for the design of rock-fall protection galleries, vulnerability assessment of concrete structures subject to corrosion degradation, assessment of the robustness of structural systems and finally it is illustrated how the framework also strongly facilitates earthquake risk management through generic indicators based hierarchical Bayesian risk models incorporated in a GIS database.

2 BASIC CONSIDERATIONS IN ENGINEERING DECISION MAKING

2.1 *Time frame for life-cycle costing and engineering*

During especially the last decades great efforts have been invested into what is now commonly understood as life-cycle costing or life-cycle engineering, see e.g. (Frangopol and Maute 2003). However, whereas there is broad agreement in regard to the consideration that the life time for engineering facilities should be taken into account in the decision making there is still large differences in what is essentially understood by the term life-cycle.

Common interpretations of life-cycle include; the investment return period of a project, the service life specified in design codes, the time until a facility becomes obsolete and the time till failure. It is clear that optimal decisions will depend on the considered time frame and it is thus important that this issue is clarified. In addressing service-life engineering it is here proposed to take basis in the duration of the purpose which a given type of structure fulfills. Considering e.g. infrastructures such as bridges and tunnels it would be fair to assume that this type of structures will not only be needed in our own generation but also in the many subsequent generations. This is already reflected in common practice by typically specifying long service lives for such structures in most modern design codes; typically code specified design lives for bridges vary between 50-200 years. However, there are other types of structures where this is not the case, such as e.g. offshore facilities utilized for exploration and production of oil and gas. For such structures common practice assumes design lives in the order of 20-50 years. This is in consistency with the fact that one given structure of this type typically operates over such short durations of time, however, neglects that any specific structure is not the last to be constructed but merely one structure in a long sequence of structures to be constructed over time in fulfillment of a general long-term function.

By considering the duration of the purpose of a given type of structure facilitates that decisions are identified which optimize the benefit achieved by types of structures rather than just the benefit from any specific structure. Within the duration of the purpose any given structure may of course fail due to extreme events and or deterioration or simply become obsolete. These events and their associated risks provide the basis for the optimization of decisions concerning design and maintenance. It also opens up for a more conscious consideration of resource usage as it automatically points to potential benefits from re-use of parts of structures and re-cycling of structural materials. In a societal context for normative decision making such as e.g. writing codes and regulations this long term perspective is important.

In Figure 1 risk based decision making is illustrated in a societal context from an intergenerational perspective; see also (Nishijima et al. 2006), (Rackwitz et al. 2005) and Faber and Maes (2007). Within each generation decisions have to be made which will not only affect the concerned generation but all subsequent generations. At an intra-generational level the characteristics of the system consist of the knowledge about the considered engineered facility and the surrounding world, the available decision alternatives and criteria (preferences) for assessing the utility associated with the different decision alternatives. A very significant part of risk based decision making in practice is concerned about the identification and representation of the characteristics of the facility and the interrelations with the surrounding world as well as the identification of acceptance criteria, possible consequences and their probabilities of occurrence. Time and space plays significant roles in this regard. Managing risks is done by "buying" physical changes of the considered facility or "buying" knowledge about the facility and the surrounding world such that the objectives of the decision making are optimized.

The important issue when a system model is developed is that it should facilitate risk assessment and risk ranking of decision alternatives which in consistency with available knowledge and which facilitates that risks may be updated according to knowledge available at future times. A system representation can be performed in terms of logically interrelated constituents at various levels of detail or scale in time and space. Constituents may be physical components, procedural processes and human activities. The appropriate level of detail and/or scale depends on the physical or procedural characteristics or any other logical entity of the considered problem as well as the spatial and temporal characteristics of consequences.

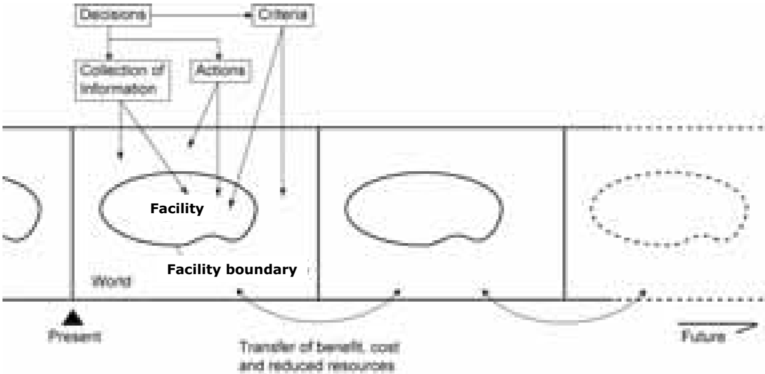

Figure 1 Main constituents of systems in risk based intra-/intergenerational decision analysis, (Nishijima et al. 2006).

2.2 *Consequences in societal decision making*

In an abstract sense risk is a characteristic of a system which indicates the potential of the system to generate consequences. For this reason it is necessary in risk based decision making to understand the details of how consequences might be generated. At a fundamental level consequences can be understood to be driven according to the second law of thermodynamics, i.e. through physical changes which all serve to maximize the entropy of the system. The effects hereof are typically material damages and associated monetary losses but also fatalities, injuries and damages to the qualities of the environment may follow. In addition information or knowledge plays an important role for consequences; knowledge facilitates taking actions and thereby to reduce risks, however information and knowledge may also trigger non-physical events associated with consequences; resource allocations forced by individual and public perception.

In Figure 2 a model framework for the assessment of consequences is proposed which address consequences due to physical changes and consequences due to perception individually. This in turn facilitates that focus can be directed to the different measures of risk reduction which are effective for the different types of consequences.

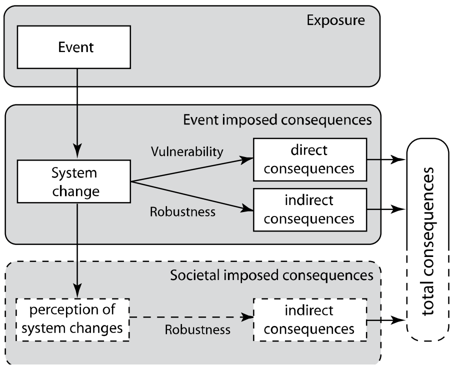

Figure 2 Representation of the mechanisms generating consequences (Faber and Maes 2007).

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Probabilities of coincidence of intermittent pulse load processes: Development of Markov methods

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ABSTRACT: Three models for intermittent pulse load processes are discussed. The first model is the train of non-overlapping rectangular pulses with Erlang-driven arrivals and truncated Erlang durations. The second one is the train of non-overlapping, non-adjoining rectangular pulses with Erlang distributed pulse durations and Erlang distributed time gaps between consecutive pulses. In the third model, the ”on/off” load process is represented as a zero-one stochastic variable governed by a stochastic differential equation driven by two independent Erlang renewal processes. For the first two models the techniques are presented to convert the original non-Markov problems into the Markov chains. The probabilities of coincidence of different, statistically independent loads are expressed in terms of the Markov state coincidence probabilities. In the third model the original non-Markov problem is converted into a non-diffusive Markov one. The equations governing directly the loads coincidence probabilities are derived.

1 INTRODUCTION

One of possible techniques of estimating the structural reliability under a number of simultaneously acting independent, intermittent, stationary loads, is evaluating an upper bound for the probability of the first excursion failure in terms of the load coincidence probabilities, Wen (1977), Winterstein (1980), Shinozuka (1981)

The Markov chain approach techniques for load coincidence probabilities were developed originally by Shinozuka (1979), Shinozuka & Tan (1979) for intermittent pulse load processes with Poisson arrivals and truncated negative exponential as well as Erlang durations. The Markov chain approach for pulses with Erlang-driven arrivals and truncated negative exponential durations was devised by Iwankiewicz & Rackwitz (1996). This approach was next generalized in Iwankiewicz & Rackwitz (1997) and in Iwankiewicz & Rackwitz (2000) to the problem of Erlang-driven arrivals and truncated Erlang distributed durations. In another model developed by Iwankiewicz & Rackwitz (1998) the intermittent pulse load process is characterized in terms of the pulse durations and time gaps between them.

In the present paper three models for intermittent pulse load processes are discussed. The first model is the train of non-overlapping rectangular pulses with Erlang-driven arrivals and truncated Erlang durations. The second one is the train of non-overlapping, non-adjoining rectangular pulses with Erlang distributed pulse durations and Erlang distributed time gaps between consecutive pulses. In the third model, the ”on/off” load process is represented as a zero-one stochastic variable governed by a stochastic differential equation driven by two independent Erlang renewal processes, Tellier & Iwankiewicz (2005b).

In the first two models, the ”on” and ”off” states of the load, determined by the Erlang renewal processes, are not Markov states. However an Erlang renewal process with an integer parameter ”k” can be expressed in terms of ”k” negative-exponential distributed variates, called ”phases” in queueing theory. Such a technique, or a ”phase approach”, allows to convert the original non-Markov problem into a chain of Markov states. Differential equations governing Markov states probabilities of a single load are derived. Based on them the differential equations for coincidence probabilities of different Markov states of various, statistically independent, loads are obtained. Finally the load coincidence probabilities are expressed in terms of Markov states coincidence probabilities.

In the third model the introduced stochastic variable only assumes values zero and one, consequently its expectation equals the probability of the load being ”on”. Thus the equation governing the mean value of this variable is identical to the equation governing the ”on” probability of the load. The state space of the system is augmented by expressing each of the driving Erlang processes in terms of auxiliary Poisson driven variables. The augmented state vector is driven by Poisson processes, hence it is a non-diffusive Markov process. The differential equations for coincidence probabilities of various , statistically independent, loads are obtained directly from the equations governing the ”on” probabilities of the individual loads.

2 MARKOV CHAIN APPROACH TO A RANDOM TRAIN OF NON-OVERLAPPING TRUNCATED RECTANGULAR PULSES

2.1 *Statement of the problem*

Consider a train of rectangular pulses whose arrival times are distributed according to a renewal process, i.e. the inter-arrival times T_a are identically distributed, positive random variables. The pulses are assumed not to overlap, i.e. each pulse duration completes before, or is truncated at, the moment of the next pulse arrival. This means that the actual duration T_d of the pulse, equals the duration T_d' sampled from the original (or primitive) distribution if the duration completes before the next arrival, and the actual duration is equal to the inter-arrival time if it has not completed until the next arrival. In the latter case two consecutive pulses adjoin each other.

The truncation scheme is expressed as

$$T_d = \begin{cases} T_d' & \text{if} \quad T_d' < T_a, \\ T_a & \text{if} \quad T_d' \geq T_a. \end{cases} \tag{1}$$

Moreover T_a and T_d' are assumed to be independent random variables. The probability density function $F_{T_d}(t)$ of the actual (truncated) pulse duration is expressed as, cf. Iwankiewicz & Rackwitz (2000)

$$g_{T_d}(t) = g_{T_d'}(t)\,(1 - F_{T_a}(t)) + g_{T_a}(t)\left(1 - F_{T_d'}(t)\right), \tag{2}$$

where $g_{T_a}(t)$ and $g_{T_d'}(t)$ are the probability density functions of the inter-arrival times T_a and primitive pulse durations T_d', respectively.

A random train of non-overlapping truncated pulses is characterized by two qualitatively different states, either the process is “off”, or it is “on”. In general these are not Markov states.

Assume that the inter-arrival times and primitive durations are Erlang distributed, with probability density functions $(t > 0)$

$$g_{T_a}(t) = (\nu k)^k t^{k-1} \exp(-\nu k t)/(k-1)!, \quad g_{T_d'}(t) = (\mu l)^l t^{l-1} \exp(-\mu l t)/(l-1)! \tag{3}$$

The problem may be converted into a Markov one by introducing a chain of Markov states. According to the ”phase approach” of the queueing theory:
– an Erlang distributed inter-arrival time is made up of k ”phases” – negative-exponential distributed variates,
– an Erlang distributed primitive duration is made up of l ”phases” – negative exponential variates,

– each phase of the inter-arrival time (arrival process) is completed in $(t, t+\Delta t)$ with probability $\nu k \Delta t$,
– each phase of the primitive duration (loading) is completed in $(t, t+\Delta t)$ with probability $\mu l \Delta t$,
– different states of the single pulse load process are determined by the coincidences of the phases of the arrival and load processes,
– the duration may be completed during any phase of the arrival process,
– due to truncation, the duration may be completed during its any phase.

Once the pertinent chain of Markov states is drawn and the transition probabilities defined, the equations governing the time evolution of Markov states probabilities

$$P_i(t) = \Pr\{S(t) = i\}, \tag{4}$$

are derived from the general expression

$$P_i(t+\Delta t) = \sum_j \Pr\{S(t+\Delta t) = i \mid S(t) = j\} P_j(t), \tag{5}$$

where $\Pr\{S(t+\Delta t) = i \mid S(t) = j\}$ is the transition probability.

2.2 *Poisson arrivals and truncated negative exponential durations (Shinozuka 1979)*

The arrival process is Poisson and the primitive durations are negative exponential distributed. The corresponding chain of two Markov states is shown in Fig. 1

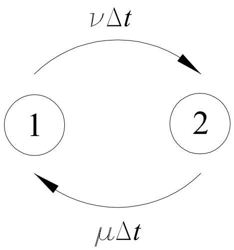

Figure 1. Markov chain corresponding to Poisson arrivals and truncated negative exponential durations.

Here $P_{off}(t) = P_1(t)$ and $P_{on}(t) = P_2(t)$.

The differential equations governing the probabilities of Markov states are

$$\begin{aligned} \dot{P}_1 &= -\nu P_1 + \mu P_2, \\ \dot{P}_2 &= \nu P_1 - \mu P_2 \end{aligned} \tag{6}$$

2.3 *Poisson arrivals and truncated Erlang durations (Shinozuka 1979)*

The arrival process is Poisson and the primitive durations are Erlang (gamma with integer parameter **l**) distributed. There is one "off" state and there are **l** "on" states, hence the number of Markov states equals $\mathbf{l} + \mathbf{1}$.

The incremental equations for Markov state probabilities $P_i(t+\Delta t)$ have to be formulated for the pertinent Markov chain and consequently the differential equations are

$$\begin{aligned} \dot{P}_1 &= -\nu P_1 + \mu l P_{l+1}, \\ \dot{P}_2 &= \nu P_1 - \mu l P_2 + \sum_{j=3}^{l+1} \nu P_j, \\ \dot{P}_i &= \mu l P_{i-1} - P_i(\nu + \mu l), \quad 2 < i \le l+1, \end{aligned} \tag{7}$$

Here $P_{off}(t) = P_1(t)$ and $P_{on}(t) = \sum_{j=2}^{l+1} P_j(t)$.

2.4 *Erlang arrivals and truncated negative exponential durations (Iwankiewicz and Rackwitz 1996)*

The arrival process is Erlang (with integer parameter **k** and the primitive durations are negative exponential distributed. There are **k** ‘off” states and the primitive duration may coincide with any of the phases of the inter-arrival time hence the total number of Markov states equals **2k**.

The incremental equations for Markov state probabilities $P_i(t + \Delta t)$ have to be formulated for the pertinent Markov chain and the resulting differential equations are

$$\begin{aligned}
\dot{P}_1 &= -\nu k P_1 + \mu P_{k+1}, \\
\dot{P}_i &= \nu k P_{i-1} - \nu k P_i + \mu P_{i+k}, \quad 1 < i \leq k, \\
\dot{P}_{k+1} &= \nu k P_k - (\nu k + \mu) P_{k+1} + \nu k P_{2k}, \\
\dot{P}_i &= \nu k P_{i-1} - (\nu k + \mu) P_i, \quad k + 1 < i \leq 2k,
\end{aligned} \tag{8}$$

Here $P_{off}(t) = \sum_{j=1}^{k} P_j(t)$ and $P_{on}(t) = \sum_{j=k+1}^{2k} P_j(t)$.

2.5 *Erlang arrivals and truncated Erlang durations (Iwankiewicz and Rackwitz 1997, 2000)*

The arrival process is Erlang (with integer parameter **k** and the primitive durations are Erlang (with integer parameter **l**) distributed. There are **k** phases of the inter-arrival time, or “off” states, and each of **l** phases of the primitive duration may coincide with any of the phases of the inter-arrival time resulting in **kl** “on” states, hence the total number of Markov states equals $\mathbf{k}(\mathbf{l} + \mathbf{1})$,

An example sequence of different possible Markov states, for $k = 2$ and $l = 2$, is shown in Figure 2. A scheme of a corresponding chain of Markov states is shown in Figure 3.

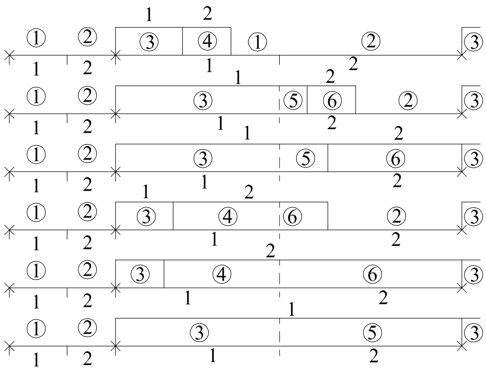

Figure 2. Sequence of different possible states for $k = 2$ and $l = 2$.

The incremental equations for Markov state probabilities $P_i(t + \Delta t)$ have to be formulated for the pertinent Markov chain (see Fig. 3) and the resulting differential equations are

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Structural reliability and existing infrastructure – Some research issues

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ABSTRACT: Structural reliability theory has made very considerable progress and in many respects can now be considered to be a mature research field. Nevertheless some serious issues remain for consideration by researchers if the theory is to find wider applications in industry, including its integration with broader approaches to risk assessment. These include the continuing need to deal with human error, the assessment of existing structures and the load and resistance models required for such assessments, the influence of load limit enforcement on bridge loads and of quality assurance on minimum material strengths and with the modelling of the deterioration of structural materials with time. An outline of the salient features of these issues is presented. It is concluded that in many cases there is a strong non-technical policy influence on each of these matters, such as for the load and resistance models that could be used for structural reliability assessment. However, this has not, to any significant extent, been applied to date. All areas discussed present significant research challenges.

1 INTRODUCTION

Structural reliability as a research area has made enormous progress since the first ideas were outlined by Mayer (1926), Freudenthal (1956) and Basler (1961) in the early-mid 1900's. Very early it was recognized that a critical difficulty was the evaluation of the multiple integrals required to estimate a probability from the multiple random variables (and later the stochastic variables) involved. A major break-through was the development of what later became known as the First Order Second Moment method. Cornell (1969), Hasofer & Lind (1974) and Rackwitz and Fiessler (1978) were key players in its development. It provided a simple and effective approach for obtaining a nominal failure probability usually expressed through a surrogate measure of reliability variously known as the safety index, reliability index, or β-index. However, it involved some drastic simplifications both of the representation of the random variables and of the physics of the structural system. Subsequent refinements include non-Second Moment random variable representation and non-linear limit state functions. These refinements all involve successive approximations and iteration. They allow the 'exact' solution to be approached asymptotically (Hohenbichler & Rackwitz 1981) but at the cost of greater computational effort. The computational requirements also increase as the order (or complexity) of the structural system increases.

Today, the availability of massive computer power means that the issue of computational requirement for time-invariant reliability problems is much less critical, and it is sometimes not easy to differentiate between the refined asymptotic methods and the conceptually much simpler Monte Carlo approaches. One advantage of the latter is that it is direct and thus easier for non-experts to code, to use and to understand. However, for time-variant problems the computational requirements can still be formidable.

The strong need to solve the computational problems associated with structural reliability estimation has meant that some of the other issues necessary to make the theory relevant to practical application and to decision-making have been somewhat slower to develop. In this context it is noteworthy that the concept of what is an acceptable level of safety (or conversely risk) has received much recent attention in the structural reliability literature (Nathwani et al. 1997, Rackwitz 2002, Ditlevsen 2004). However it is fair to say that these developments do not appear always to associate closely with the very extensive work done on this issue in other, quite disparate, fields of research and practical application. This, includes the management of risks for the nuclear and petro-chemical industries (Conrad 1980) and the allocation of scarce resources for humanitarian aid (Dasgupta 2002). Nevertheless, there can be no doubt that greater unity in these areas will develop. From a global perspective the methodology ought to be the same. Moreover, it can be argued that there should be consistency across all areas dealing with risk assessment, including the estimation of hazards and associated probability of occurrence, the hazard consequences and the societal acceptance of both, even though currently there are significant differences of approach (Melchers and Stewart 1997). This still represents a significant challenge, not only for structural reliability researchers.

The above suggests that some important issues remain for the further development of structural reliability theory and in particular its practical application. These include how to deal with human error, the assessment of existing structures rather than the design-code requirements for new ones and the modelling of loads and resistances in such a way to represent reality in the crucial tail regions. These last are controlled, ultimately, by societal expectations rather than engineering criteria. A rather subjective view of these matters is presented below, together with several pointers to potential research directions.

2 HUMAN ERROR

As has been pointed out time and again, in practice most failures of structures result from some sort of human error (Chou 2005). Early efforts to categorize human error ultimately failed to provide useful practical courses of action and outcomes, as did efforts to obtain detailed data on the rate and form of errors in design and construction (Stewart & Melchers 1997). More recent research has concentrated on organizational issues and how this might relate to good design and construction outcomes (Blockley & Godfrey 2000, van der Molen 2006). The implication is that this will lead to safer and more serviceable structures. Also, the role of quality assurance processes in material quality, design processes and in construction has been much canvassed. However, it is difficult to see that these have much impact on gross errors, accidentally or otherwise, and neglect. Overly optimistic assessments, poor estimation or ignorance of known or potential loadings often is seen to be the key issue in structural failures of modern structures in societies with well-developed and well-implemented building control systems. In contrast, poor quality, illegal or corrupt workmanship and construction practices often are a feature of structural failures in societies with economic and perhaps social difficulties, irrespective of the quality of the engineering services available. Evidently, these are primarily social and economic issues rather than engineering ones

A key aspect is the poor link between the lack of achieved quality in construction, and by implication in theoretical structural safety, and actual outcomes. Typically the loadings used in the design process are largely nominal, idealized and conservative. They are only seldom realized or even approached during the actual lifetime of the structure. There can be no doubt that in practice many structures appear be perfectly satisfactory and viable even though rather serious errors in design and/or construction have been built into them. Only exposure to the full design loads and the various load combinations would reveal the deficiencies but for most structures such load scenarios rarely occur if they occur at all during the lifetime of the structure. As a result, the chances of defects in design or construction having an actual outcome in structural failure are very low. Undoubtedly this observation has not escaped those tempted by economic considerations to take 'short-cuts' in design and construction. It is also one of the issues with the selling-on of apartment buildings immediately after the guarantee period and before construction defects have had time to become evident. Again, this is a sociological issue.

Social, political and moral matters are notoriously difficult to formulate in a quantitative theoretical sense (Luthans 1973) and do not readily integrate with engineering models. This suggests that they are not matters for structural reliability theorists to build into their models. Comparable issues arise in all forms of human endeavour where safety is involved. The crucial matter is how much control and/or enforcement society is willing to apply. Ultimately this depends on the (fickle) political climate and on societal perceptions and expectations, irrespective of whether they are in any sense logical. All structural engineers can do (and certainly should do) is to remind of the risks involved in their activities and also to remind that those risks are incurred on behalf of society, either directly or indirectly. Society and its agents must deal with them (Melchers 2001).

3 ASSESSMENT OF EXISTING STRUCTURES

The assessment of existing structures requires the availability of appropriate assessment tools and techniques. It also requires models of loadings and of resistances that reflect, sufficiently accurately, the behaviour and condition of the existing structure. Importantly, these models will be expected to differ from the models used for design. A crucial difference is that the design models need to deal with an as yet un-built structure. They need to represent the structure expected to result from the design and construction process in some notional way. Invariably the uncertainty about the prospective structure and its characteristics and properties will be much greater at design time than after the structure has actually been realized. On this basis alone it can be expected that the models used for assessment will be considerably different from those used conventionally for the design of new structures.

For the design of new structures it has been traditional to employ simple but conservative design tools and procedures. One reason is that since not everything can be predicted accurately at the time of design, overly accurate methods simply are not justified, in general. Long experience indicates that this is an acceptable way of proceeding for most structures, with the associated economic penalties assumed balanced by savings in design efforts. Typically only prestige, very expensive structures warrant more detailed modelling and design effort.

For the assessment of existing structures simple and conservative assessment processes might be useful for indicating whether there is a potential problem. However, as decision tools for assessment they can lead to unnecessary repairs or retro-fitting or condemnation for apparent failure to meet structural safety or serviceability criteria based on new designs. It has been observed repeatedly that application of current design rules to existing systems often predicts imminent failure, even if in practice it is abundantly clear that the structure is still quite satisfactory.

The reasons why it is inappropriate to employ design code rules (i.e. the rules for the design of new structures) for the assessment of existing structures include the following:

1. the nominal loadings used in the design process may not be representative of the actual loadings for the 'as-built' structure, both during its current and its expected future operation,

2. the nominal loadings used in the design process are unlikely to be achieved during the actual lifetime of the structure,
3. design rules make allowances, in the design models, for uncertainties in the strength that is expected to be achieved in eventual construction whereas these uncertainties are largely realized in an existing structure, that is, they are no longer as uncertain (i.e. apart from measuring them), and
4. the actual, rather than the expected, deterioration of the structure, such as through fatigue or corrosion, must be allowed for in assessment, as must possible increases in the aggressiveness of the environment.

Unfortunately, proper understanding of the future actual behaviour and safety of an existing structure is not necessarily directly derivable from the usual code rules, or from the models and analysis procedures typically employed for the design of new structures. Such modelling tends to make assumptions about the expected structure and thus may not represent the reality of the existing structure. In contrast, for assessment the reality is much more important. It follows that more emphasis will need to be given to developing better understanding of the behaviour of existing and older structures, and the loading conditions under which they operate in actual practice. Clearly, this opens up a very broad and challenging research field, as yet with only few researchers. Perhaps the full impact of infrastructure deterioration and the economic drivers for deferment of remedial action have not yet been sufficiently keenly felt in practice.

4 THE SAFETY OF EXISTING STRUCTURES

The structural safety and technical reliability of a constructed structure will reduce as the structure becomes older and receives greater usage. Figure 1 shows this schematically, assuming that the design and construction processes do not reduce reliability and ignoring small increases in safety that may arise from slow increases in strength, such as in concrete compressive strength.

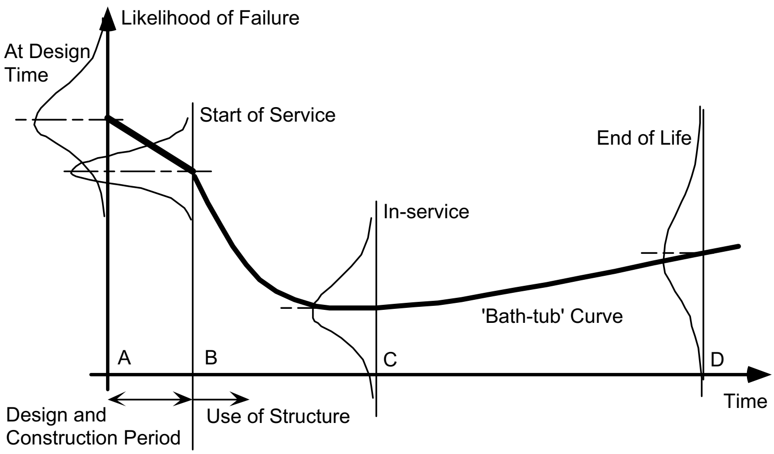

Figure 1. Typical variation of failure likelihood and uncertainty as a function of time.

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Application of the Life-Quality Index to optimize infrastructure maintenance policies

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ABSTRACT: Aging and degradation of infrastructure facilities and systems have raised concerns over the safety of public, environment and financial investments. Large investments are required to upgrade civil and industrial infrastructures to sustain the economic well being and quality of life in the society. The paper illustrates an application of the Life Quality Index (LQI) to assess the effectiveness of infrastructure renewal projects that have major impact on life safety.

1 INTRODUCTION

The safe and efficient management of engineering infrastructure systems, such as power plants, pipelines, transmission lines, bridges, highways, water distribution and waste-disposal systems, directly contributes to economic well being and quality of life in the society. Deterioration of such infrastructure facilities built during 1960s and 70s has raised concerns over safety of the public and the environment. Since large investments are required to upgrade civil and industrial infrastructure, improved models are needed to optimize strategies for managing the refurbishment and life extension of existing infrastructure systems.

The objective in managing risk is to ensure that significant risks are identified and appropriate actions are taken to minimize these risks to a reasonably low level. Engineered safety is determined on the basis of a balance between the cost effectiveness of risk control and the benefits arising from the mitigation of risk. For the net benefit to be positive, whether it accrues to the organization or society at large, the management of risk entails a process of priority setting because there are limits on available resources.

The Life Quality Index (LQI) provides a new approach to managing risk, which goes beyond the traditional focus of economic loss mitigation into an arena of risk to life and health (Pandey et al. 2006). The LQI is an ordinal utility function that quantifies the utility of income derived over the expecte lifetime of a representative individual in the society. It comprises of economic, demographic and life-safety aspects of a society. Professor Rackwitz has provided much more rigorous footing to LQI and presented its applications to several practical examples (Rackwitz 2002, 2003, Rackwitz et al. 2005).

The paper illustrates that the societal capacity to commit resources for risk reduction in a sustainable manner can be derived from the LQI. The paper presents an exposition of the LQI-based benefit-cost analysis method. It clarifies the underlying concepts, computational procedures and provides the interpretation of results so that engineers can apply this method to practical examples of infrastructure management.

2 LIFE QUALITY INDEX

2.1 *General Concept*

LQI is an ordinal utility function that quantifies the utility of income derived over the potential lifetime of a representative individual in the society (Pandey et al. 2003, 2006). The LQI has been derived as

$$L = cg^q e \tag{1}$$

where *e, g* and *c* are the life expectancy, real Gross Domestic Product per person and a constant, respectively. The parameter *q* has especial significance as it reflects a trade-off that the society places between economic consumption and the value of the length of life. Using macroeconomics theories, the exponent was derived as

$$q = \frac{1}{\beta}\frac{w}{(1-w)} \tag{2}$$

In this expression, β denotes the share of labour input to GDP and *w* is the work time fraction in a year. Based on economic time series data, it was shown that *q* exhibits small fluctuation over time, and the mean value of $q = 0.2$ was recommended for practical applications. The fact that $q < 1$ reminds that the money is an imperfect substitute for lifetime A formal derivation, interpretation and calibration of the LQI have been presented elsewhere (Pandey et al. 2006). The constant *c* is not relevant to cost-benefit analysis because of its incremental nature.

The calculation of LQI can be illustrated using some practical data. The life expectancy at birth in Canada for example is 77.5 years. Assuming a value of the real GDP per capita as g = 30,000 \$/person/year, the LQI is computed as $(30{,}000)^{0.2} \times (77.5) = 609$ *utils*.

2.2 *Life Expectancy and Related Concepts*

Define the probability density function of the lifetime, *T*, as $f_T(t)$, and use a concise notation to denote it as $f(t)$. In general, the life expectancy at birth is defined as

$$e = \int_0^{a_u} t f(t)dt = \int_0^{a_u} S(t)dt$$

where $S(a)$ is the probability of survival up to age *a*, which can be defined in terms of the lifetime density and mortality rate, $m(t)$, as

$$S(a) = \int_a^{a_u} f(t)dt = \exp\left[-\int_0^a m(\tau)d\tau\right] \tag{3}$$

where a_u is some maximum value of the human lifetime (≈110 years). Survival probabilities for different ages are described in an actuarial life table for a country. The current survival and hazard (or mortality) curves for Canada are shown in Figure 1.

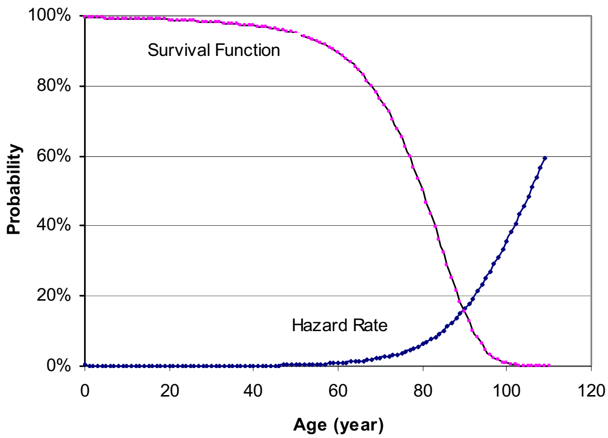

Figure 1: Survival function and hazard rate of the human lifetime

The life expectancy changes with the age of the person. To illustrate this, the conditional probability density function of the lifetime of a person surviving up to age a is introduced as

$$f_T(t \mid T > a) = \frac{f(t)}{P[T > a]} = \frac{f(t)}{S(a)} \tag{4}$$

The remaining life expectancy of a person of age a is denoted as $e(a) = E[T\text{-}a|T>a]$.

$$e(a) = E[T - a \mid T > a] = \int_a^{a_u} (t-a) f(t \mid T > a)\, dt = \int_a^{a_u} (t-a) \frac{f(t)}{S(a)} dt = \int_a^{a_u} \frac{S(t)}{S(a)} dt \tag{5}$$

The ratio of survival probabilities in Eq.(5) can be expressed in terms of the mortality rate as

$$\frac{S(t)}{S(a)} = \frac{\exp\left[-\int_0^t m(\tau) d\tau\right]}{\exp\left[-\int_0^a m(\tau) d\tau\right]} = \exp\left[-\int_a^t m(\tau) d\tau\right], \quad 0 \le a \le t \tag{6}$$

Substituting Eq. (6) into (5) leads to

$$e(a) = \int_a^{a_u} \exp\left[-\int_a^t [m(\tau)] d\tau\right] dt \tag{7}$$

If the mortality rate is changed from $m(x)$ to $[m(x) + h(x)]$, it would modify the lifetime distribution. The modified distribution can be denoted by a new random variable T_1 and the mean lifetime can be obtained as

$$e_1(a) = \int_a^{a_u} \exp\left[-\int_a^t [m(x) + h(x)] dx\right] dt \tag{8}$$

The change in life expectancy is the average change in lifetime estimated as $de = E[T - T_1] = (e - e_1)$. It should be noted that a change in mortality rate at any age $t \ge a$ will influence the remaining life expectancy, and the change in life expectancy is an average quantity that occurs over the lifetime of an individual.

2.3 *Benefit-Cost Analysis using LQI*

One important goal in managing risks to life safety is to determine an acceptable level of expenditure that can be justified on behalf of the public in exchange for a small reduction in the risk of death without comprising the life-quality. It can also be referred to as the societal capacity to commit resources (SCCR) and it can be obtained from the LQI invariance criterion as follows.

Suppose that a risk management program has a potential to improve the life expectancy from a reference or baseline level of e to (e+ de). A threshold value of the cost of the program, dg_L (in \$/year), can be calculated from the invariance criterion such that LQI in the reference case (g, e) is the same as in the new scenario (g-dg_L, e+ de), as shown in Figure 2. This condition can be expressed as

$$\frac{dL}{L} = 0 \Leftrightarrow \frac{dg_L}{g} + \frac{1}{q}\frac{de}{e} = 0 \tag{9}$$

A threshold value of the cost rate can thus be derived as

$$(-dg_L) = \frac{g}{q}\frac{de}{e} \qquad (\$/\text{year}) \tag{10}$$

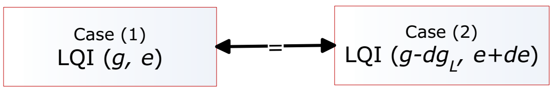

Figure 2: LQI invariance principle

A careful interpretation of all the terms in Eq.(10) is important. If the risk management program results in a gain in life expectancy, dg_L represents the maximum allowable cost rate to fund the program. In other words, if the actual program cost rate is less than dg_L, the program improves the LQI for the population under consideration. If a project results in loss of life expectancy, then dg_L represents minimum benefit that should be derived from the project. Otherwise, it will result in a decrease in the LQI.

The term de/e is in the unit of life years gained (or lost) per year of lifetime. The maximum cost of saving one life year can then be computed from Eq.(10) as

$$\frac{(-dg_L)}{de/e} = \frac{g}{q} \qquad \{(\$/\text{year})/(\text{life year/year}) = \$/\text{life year}\} \tag{11}$$

For g = 30,000 and q = 0.2, this value is 150,000 \$/life year saved. This value is independent of the age of the person.

Another interesting quantity is dg/de, which is interpreted as the cost rate in \$/year for saving one life year:

$$\frac{(-dg_L)}{de} = \frac{1}{q}\frac{g}{e} \qquad (\$/\text{year/life year}) \tag{12}$$

In summary, so long as the cost rate of implementing a program is less than that given by eqn.(10), the program can be considered to be beneficial from the LQI point of view.

If the benefits of a safety project accrue to a population of size N, then the aggregated value of justified expense, i.e. the amount that will not alter the population life-quality, is the societal capacity to commit resources (SCCR) and given as

$$\text{SCCR} = (-dg_L)N = N\frac{g}{q}\frac{de}{e} \quad (\$/\text{year}) \tag{13}$$

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Structural reliability aspects in design of wind turbines

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ABSTRACT: Reliability assessment, optimal design and optimal operation and maintenance of wind turbines are an area of significant interest for the fast growing wind turbine industry for sustainable production of energy. Offshore wind turbines in wind farms give special problems due to wake effects inside the farm. Reliability analysis and optimization of wind turbines require that the special conditions for wind turbine operation are taken into account. Control of the blades implies load reductions for large wind speeds and parking for high wind speeds. In this paper basic structural failure modes for wind turbines are described. Further, aspects are presented related to reliability-based optimization of wind turbines, assessment of optimal reliability level and operation and maintenance.

1 INTRODUCTION

The number and size of wind turbines are increasing fast these years. New typical wind turbines are 3-5 MW turbines with hub height 80-110 m and rotor diameter 100-150m. Offshore wind turbines are placed in wind farms with 50-100 wind turbines and loadings due to both wind, wave, current and ice besides harsh environmental attack implying corrosion etc. Onshore wind turbines are more and more placed in complex terrain with good wind conditions for energy production. Reliability analysis and optimization of wind turbines require that the special conditions for wind turbine operation are taken into account. Control of the blades implies load reductions for large wind speeds and parking for high wind speeds.

Optimal design of wind turbines should be obtained using a life-cycle approach where all relevant benefits and costs are included. Formulation of the associated general reliability-based optimization problem with a maximum acceptable probability of failure constraint is considered in this paper. Offshore wind turbines are characterized by a low risk of human injury in case of failure when compared to onshore wind turbines, and to civil engineering structures in general. It is then relevant to assess the minimum reliability level for the structural design on the basis of reliability-based cost optimization considering the whole life-cycle of the turbines without a reliability constraint. Especially for offshore wind turbines costs related to operation and maintenance can be significant and have to be included. One reason is that maintenance can only be performed under certain weather conditions.

Behind a wind turbine a wake is formed where the mean wind speed decreases and the turbulence intensity increases, (Frandsen 2005). The distance between the turbines is among other things dependent on the recovery of wind energy behind the neighboring turbines and the increased wind load. This paper describes the basic relationships of mean wind speed and turbulence intensity in wind turbine parks with emphasis on modeling the wind load. For offshore wind turbines also environmental loads from waves, ice and current can be significant.

The basic structural failure modes for wind turbines are fatigue failure in the tower – typically in welded steel connections; in the blades made of glass fiber and/or composites; and in the nacelle made of cast steel. Ultimate failure due to extreme load has to be considered in the tower, the blades, the nacelle and the foundation. The wind load effects depend highly on the chosen type of control (stall or pitch) in the operational mode, i.e. for mean wind speeds at hub height below 25 m/s. Also non-structural failure modes such as failure of electrical components and machine components e.g. gear boxes have to be considered because they influence the load on other structural components.

This paper describe some aspects for reliability analysis by structural reliability methods, see e.g. (Rackwitz 2002a) and optimization of wind turbines, including wind loads in wind farms, ultimate and fatigue failure modes, optimal reliability level and operation and maintenance.

2 WIND LOAD

For wind turbines in a wind farm the turbulence is increased significantly behind other wind turbines, and the wind velocity is decreased slightly. Figure 1 shows a typical layout of a small wind farm. In figure 2 from (Frandsen, 2005) data are shown obtained from this wind farm. It is seen that the wind load effect (flapwise blade bending moment) proportional to the standard deviation of the turbulence increase significantly behind other wind turbines.

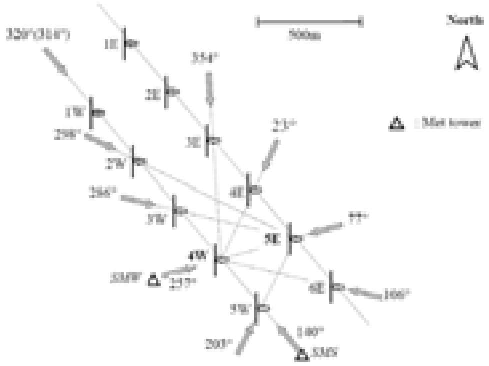

Figure 1. Layout of the Vindeby offshore wind farm - from (Frandsen, 2005).

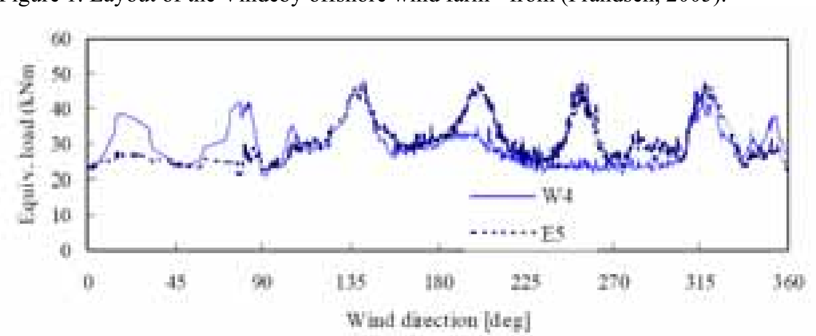

Figure 2. Equivalent load effect (flap-wise bending) for two wind turbines, 4W and 5E in figure 4, as function of wind direction, in the Vindeby wind farm, $8 < U < 9$ m/s - from (Frandsen, 2005).

As mentioned in the introduction, the operational mode of a wind turbine is important when modeling the wind load. The wind turbine is in *standstill* mode if the 10-minutes mean wind speed at hub height exceeds 25 m/s, and the wind load corresponds to the annual extreme wind load. At lower wind speeds the wind turbine is in the *operational* mode and produces electricity. The wind velocity is at maximum 25 m/s. The maximum wind load is dependent on the control system (stall or pitch) of the wind turbine and the maximum turbulence intensity. Since the number of 10-minutes periods with wind velocity at the critical velocity smaller than or equal to the 25 m/s is in general large, the maximum wind load during operation is obtained from simulations of the wind velocity field in front of the wind turbine and the actual control system of the wind turbine. A distribution function is determined for the maximum wind load over the expected number of 10-minutes periods with wind velocity from approximately 10 m/s to 25 m/s.

3 LIMIT STATE EQUATIONS FOR STRUCTURAL RELIABILITY ASSESSMENT

3.1 *Probabilistic model for ULS failure*

For the typical structural failure mode 'bending failure of the tower' the limit state equation for a single wind turbine in *standstill mode* can be written, see (Tarp-Johansen et al. 2003) and (Sørensen and Tarp-Johansen 2005):

$$g_S(z_S, \mathbf{X}) = R(z_S) - Qh \tag{1}$$

where h is the hub height and the bending moment resistance is $R = z_S X_R F_y$. z_S is the design variable in standstill mode (e.g. section modulus), F_y is the yield strength and X_R is the model uncertainty. The load effect is

$$Q = P\, C_T A\left(1 + 2k_p I c_{amp} X_{dyn}\right) X_{aero} X_{\exp} X_{st} X_{str} X_{sim} \tag{2}$$

where $P = 0.5\rho V^2$ is the extreme mean wind pressure , U is the extreme annual mean wind velocity at hub height, $I = \sigma_u / V$ is the turbulence intensity at hub height, σ_u is the standard deviation of turbulence at hub height with standard deviation $\sigma[\sigma_u] = 2I_{15}$ and expected value $E[\sigma_u] = \hat{\sigma} - \sigma[\sigma_u]$, (Tarp-Johansen et al. 2003). I_{15} is the turbulence intensity corresponding to a mean wind speed equal to 15 m/s. The characteristic value $\hat{\sigma} = I_{15}(15 + aV)/(1+a)$ is assumed to be a 90% quantile. All parameters are described in table 1.

In *operational mode*, the wind turbine is operating at a wind velocity which is maximum equal to 25 m/s. The limit state equation is written:

$$g_O(z_O, \mathbf{X}) = R(z_O) - Qh \tag{3}$$

where the bending moment resistance is $R = z_O X_R F_y$ and the load effect is

$$Q = P\, C_T A\left(\eta + (1-\eta) T \frac{\sigma_u}{E[\sigma_u]} X_{dyn} X_{St} X_{ext} X_{sim}\right) X_{aero} X_{\exp} X_{str} \tag{4}$$

z_O is the design variable in operation mode, η is a parameter modeling the mean response relative to the expected extreme response, $P = 0.5\rho V^2$ is the mean wind pressure at maximum operational wind velocity V = 25 m/s, T is the normalized annual extreme load during operation, which is assumed to be Gumbel distributed, see (Tarp-Johansen et al. 2002) and (Tarp-Johansen et al. 2003). All parameters are described in table 1.

Table 1. Example stochastic variables for local buckling failure mode. Variables denoted X model model-uncertainty. N: Normal, LN: Lognormal, G: Gumbel.

Variable		Distribution type	Mean value	*COV*	Characteristic value
h	Rotor height		70 m		
P	Annual maximum mean wind pressure - Standstill	G	538 kPa	0.23	98%
P	Operation mean wind pressure - Operation		$0.5\rho(25\text{m/s})^2$		
T	Annual maximum normalized operational wind load – Operation	G	$E[T]$	$COV[T]$	98%
σ_u	Standard deviation of turbulence - Standstill	LN			90%
σ_u	Standard deviation of turbulence - Operation	LN			90%
X_{wake}	Model uncertainty	LN	1	0.15	1
$C_T A$	Thrust coefficient × rotor disk area				
c_{amp}	Factor		1.35		
k_p	Peak factor		3.3		
$X_{\exp}$	Exposure (terrain)	LN	1	0.10	
X_{st}	Climate statistics	LN	1	0.05	1
X_{dyn}	Structural dynamics	LN	1	0.05	1
X_{aero}	Shape factor / model scale	G	1	0.10	1
X_{sim}	Simulation statistics – Standstill	N	1	0.05	1
X_{sim}	Simulation statistics - Operation	N	1	0.05	1
X_{ext}	Extrapolation – Operation	LN	1	0.05	1
X_{str}	Stress evaluation	LN	1	0.03	1
F_y	Yield stress	LN	240 MPa	0.05	5%
X_M	Resistance – model uncertainty	LN	1	0.03	1

In a *wind farm* the limit state equation for a wind turbine in *standstill* is the same as for a single wind turbine. In *operational mode* wakes behind wind turbines change the wind flow, see above. In the following it is assumed conservatively that the mean wind velocity is unchanged, and that the standard deviation of the turbulence is increased basically following the models recommended in (IEC 61400 2005). The limit state equation is written:

$$g_O(z_O, \mathbf{X}) = R(z_O) - Qh \tag{5}$$

where the resistance is $R = z_O X_R F_y$ and the load effect is

$$Q = P\, C_T A\left(\eta + (1-\eta) T X_{dyn} X_{St} X_{ext} X_{sim} \sigma_u / E[\sigma_u]\right) X_{aero} X_{\exp} X_{str} \tag{6}$$

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Fatigue modelling according to the JCSS Probabilistic Model Code

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ABSTRACT: The Joint Committee on Structural Safety is working on a Model Code for full probabilistic design. The code consists out of three major parts: Basis of design, Load Models and Models for Material and Structural Properties. The code is intended as the operational counter part of codes like ISO, Eurocodes and other national codes, that allow for probabilistic design but do not give any detailed guidance. This publication will give a summary of the section on fatigue modelling, which is presently under development. Special attention is given to the spatial correlations, which may be of particular importance in setting up and interpreting inspection results.

1 INTRODUCTION

The present ISO 2394 document on Reliability of Structures considers the Partial Factor Method and the Full Probabilistic Methods as equivalent procedures for the verification of structural reliability. Also the Eurocode EN1990, Basis of Design, offers the member states of the European Union the possibility to accept probabilistic methods as a valid alternative procedure for the verification of structural reliability.
In practice, however, only the partial factor method is directly operational as it is supported by a set of action and material codes that provide adequate information on calculation models, characteristic values, partial factor values, load combinations and so on. At this moment the full probabilistic procedures clearly lacks such a follow up. The resulting difficulties for the designer are clear. In the first place he has to do an extensive literature research in order to find out available data and models. More important, however, is that this way the result will always depend to a certain degree on the person who performs the analysis. As a consequence probabilistic analysis becomes unacceptable for most authorities.
Having this in mind, the Joint Committee on Structural Safety of IABSE, CIB, RILEM, *fib* and ECCS have decided a couple of years ago to write a Probabilistic Model Code. This code is now available on the Internet (see http://www.jcss.ethz.ch). It should be noted that the code is not intended as a textbook on structural reliability engineering. The code is written in a condensed way and little or no educational explanations are given. The code is also a "living document" as new parts are added almost every year. Presently the JCSS is working on the extension for fatigue modelling, which however proves to be a difficult task. This paper gives the present state of affairs.

2 JCSS PROBABILISTIC MODEL CODE

The first part of the JCSS Probabilistic Model Code is concerned with the Basis of Design. It is based on the work by Ditlevsen and Madsen (1989) and Ditlevsen (1991) as well as on the present version of ISO 2394 mentioned before. The main topics of this part are the general requirements for structural reliability, the limit state concepts, uncertainty modelling, and last but not least target reliabilities (Rackwitz, 2000 and 2001) and the Bayesian interpretation of probabilistic analysis.
The second part of the JCSS Model Code presents the principles of load modelling and a number of sections describing probabilistic models for individual loads. Loads that are presently described are self-weight, live load, wind, snow, traffic, car parks, earthquake, impact and fire. In principle, for all loads random time and spatial variability is considered. As far as possible, the modelling is based on (see Figure 1) simple homogeneous Ferry Borges Castanheta or FBC models (Ferry Borges, 1971), Gaussian of Poisson models and hierarchical combinations of them (Rackwitz, 1992).
The resistance part of the JCSS Probabilistic Model Code treats materials like concrete, reinforcement steel, structural steel, timber and soil. The part further presents information on probabilistic models for cross sectional dimensions, imperfections (in particular eccentricities for the buckling of columns) and model uncertainties. Material properties are defined as the properties of material specimens of defined size and conditioning, sampled according to given rules, subjected to an agreed testing procedure, the results of which are evaluated according to specified procedures. The main characteristics of the mechanical behaviour are described by the one dimensional σ-ε-diagram, but also other properties as multi-axial stress behaviour, duration and rate effects and response to physical and chemical influences are of importance.

Also for material properties attention is given to the spatial variations in the same way as for the loads. Three hierarchical levels of spatial variation are considered (Kersken Bradley and Rackwitz, 1991): the macro (global or structural) level, the meso level (test specimen or unit, structural member) and micro (material) level. The micro scale is assumed to give guidance on the type of distribution (e.g. Weibull for brittle materials). On the meso scale a member or structure is regarded as being constituted from a sequence reference test volumes or units. Such a sequence can be described by a fixed set of statistical parameters (mean, standard deviation) The values within in one member have to be considered as being correlated, with a coefficient of correlation depending on the distance Δr. The correlation between the values for different members usually is modelled by a single parameter for a total structure, regardless the distance. At the highest macro scale level, we have the total country or worldwide production of a material, which in its turn can be conceived as a sequence of lots.

3 THE JCSS MODEL CODE FATIGUE MODEL

3.1 *One dimensional crack growth model*

The crack growth model for fatigue proposed to be adopted in the Model Code is the bi-linear version (see Figure 1) of the model by Paris & Erdogan (1963):

$$\frac{da}{dN} = C_1(\Delta K)^{m_1} \quad \text{for} \quad \Delta K_{th} < \Delta K < \Delta K_{tr} \tag{1}$$

$$\frac{da}{dN} = C_2(\Delta K)^{m_2} \quad \text{for} \quad \Delta K_{tr} < \Delta K \tag{2}$$

where a is the crack depth, da/dn is the instantaneous crack propagation rate and ΔK the alternating stress intensity factor at the crack tip (see equation (3)), ΔK_{th}, a cut off value below which the crack is assumed to be non-propagating; the parameters C_i and m_i are material

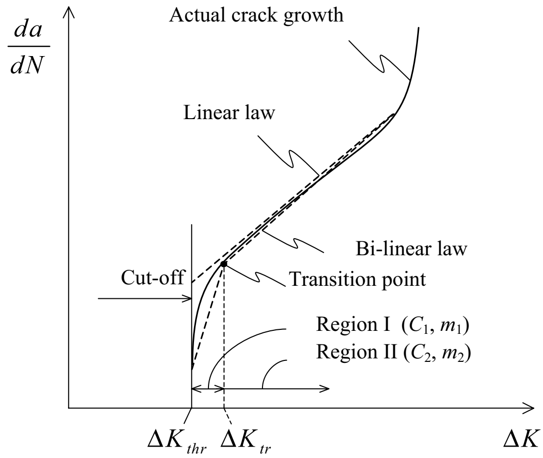

Figure 1. Schematic representation of typical crack growth rates with linear and bi-linear approximations.

constants which can be determined from experiments. Finally ΔK_{tr} is the value of ΔK at the transition point between the crack growth curves (1) and (2).

The stress intensity factor K for a given applied 'far field' fluctuating stress is given as:

$$K = S_a \, U_a \, Y \, M_k \sqrt{\pi \, a} \tag{3}$$

The stress should be determined near the crack location (but not influenced by the crack or the details of the weld). It includes, however, the stress concentration, resulting from the global geometry of the joint. The geometry factor Y reflects the local geometry of the joint in the vicinity of the crack and the nature of stress distribution. The factor M_k accounts for the effect of a crack being in the immediate vicinity of a weld, a notch or a hole. U_a incorporates the effects of residual stresses and stress ratio R_S. For details on Y, U and M, see (JCSS, 2007). Further refinement is possible by making a distinction between stresses resulting from bending and membrane action.

3.2 *Two dimensional crack growth model*

For welded joints, micro-cracks often initiate from surface-breaking defects at the toe of the weld. These micro-cracks tend to coalesce to form a single, dominant fatigue crack of roughly semi-elliptical shape. Hence, semi-elliptical cracks in plated structures are of interest in many practical applications (Figure 2). In this case, two crack dimensions, the depth a and the half-length at the surface $c,$ become relevant both of which are functions of the fatigue loading process.

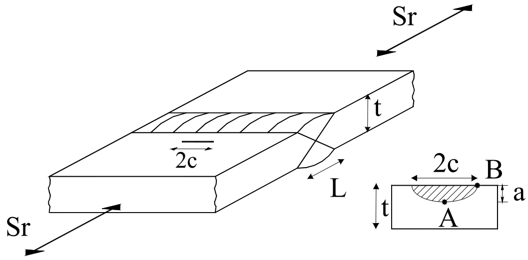

Figure 2. A semi-elliptical crack in a steel plate at/near the weld toe.

For each principal direction of crack growth a Paris type expression may be formulated as (linear case for convenience):

$$\frac{da}{dn} = C(\Delta K_a)^m \quad \text{for } \Delta K_a > \Delta K_{thr} \tag{4}$$

$$\frac{dc}{dn} = C(\Delta K_c)^m \quad \text{for } \Delta K_a > \Delta K_{thr} \tag{5}$$

where the first expression relates to point A (growth in the depth direction) and the second expression relates to point B (growth in the length direction). For each of these points, the stress intensity factor range, ΔK_a and ΔK_c, is given by:

$$\Delta K_a = \Delta S\, U_a Y_a M_{ka} \sqrt{\pi\, a/Q} \tag{6}$$

$$\Delta K_c = \Delta S\; U_c Y_c M_{kc} \sqrt{\pi\, a/Q} \tag{7}$$

where S is the applied stress range, Y_a, Y_c are geometry factors and M_{ka}, M_{kc} are stress magnification factors for points A and C respectively, and Q is the elliptic shape factor (Newman & Raju, 1986) which for $a<c$ may be approximated by $Q = 1+1.46\ (a/c)^{1.65}$. After substituting (6) and (7) into (4) and (5) a pair of coupled differential equations is obtained. In general, an incremental numerical procedure is used, which involves sub-division of the total time into a number of steps. In the bilinear case the stats of *a* (being in the first or second branch of the crack growth model) also determines the status of the process in c-direction.

3.3 *Limit state formulations*

The limit state function for a crack through failure may be formulated as:

$$g_1(\mathbf{X}, t) = a_c - a(t) \tag{8}$$

where a_c is a limiting crack depth (e.g. the plate thickness) and $a(t)$ is the crack depth after a service exposure of time *t.* At t=0 the crack size has an initial value a_i and it increases as a function of time according to the process described in the sections 3.1 and 3.2. The vector of random variables **X** comprises the parameters a_i, C_1, C_2, ΔK_{tr} etc. The calculation of the crack size at time *t* is not trivial as the stress $S(t)$ is a random process. We will revisit this issue in section 5.

Historical development and special aspects of reliability theory concepts in design codes

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ABSTRACT: This article illustrates the development of the German design codes, particularly for structural concrete construction. Special attention is given to the historical influence on the German concrete construction code caused by the emerging developments in the field of reliability theory. Furthermore, the currently implemented reliability concept will be described briefly and the resulting problems concerning the design of individual structural components will be elucidated. As a concluding remark, remaining unresolved issues from an engineer's point of view are discussed.

1 INTRODUCTION

In the last decades, the international committees of construction engineering worked on the implementation of the latest scientific findings on probabilistic theory in practical design procedures. This resulted in a set of guidelines, reports on the state of the technology etc., which were incorporated into the ongoing development of national and international codes. The significant step towards a harmonization and expansion of design codes in the field of construction engineering was initiated by the Commission of European Community through the establishment of the Eurocode. Henceforth, these Eurocodes are to be obligatorily introduced shortly (presently intended in 2010). However, the ubiquitous common question as to how significant the probabilistic error is still remains.

2 DEVELOPMENT OF STRUCTURAL DESIGN

2.1 *Concrete Structures*

Historically, at the outset of reinforced concrete construction standards in the German empire and other countries, there was an aspiration for verification of the reliability of concrete structures, consequently followed by the attempt to compile recommendations, regulations and engineering standards. These attempts passed several stages and on the 16th of April in 1904 the Prussian Governmental Department for Public Works published the "regulations for the construction of buildings made of ferro concrete" (PGDPW 1904), which marked the beginning of concrete construction standards in Germany.

These "regulations" as shown in Figure 1 comprised merely ten pages, whereas the third section with the caption "method of calculation with examples" covered more than half of the volume. However, a glance at the table of contents shows that the range of regulations, including support structure planning (section II. and III.), the testing of construction material and the final on-site building inspection (section I.), is by all means comparable with modern regulations, albeit the extents of the regulations were comparatively modest due to the limited experience at that time. The guidelines for structural analysis in section II are limited to the design for bending

Bestimmungen für die Ausführung von Konstruktionen aus Eisenbeton bei Hochbauten

I. Allgemeine Vorschriften
- A. Prüfung
- B. Ausführung
- C. Abnahme

II. Leitsätze für die statische Berechnung
- A. Eigengewicht
- B. Ermittlung der äußeren Kräfte
- C. Ermittlung der inneren Kräfte
- D. Zulässige Spannungen

III. Rechnungsverfahren mit Beispielen
- A. Reine Biegung
- B. Zentrischer Druck
- C. Exzentrischer Druck
- D. Beispiele

Figure 1. Table of content of the Prussian "regulations" from (PGDPW 1904).

with and without normal force, as well as shear force. The determination of the internal forces and the structural design were based on the theory of elasticity, whereas the ratio of the moduli of elasticity of iron to concrete was given at n = 15. In conclusion, it can be stated that the "regulations" of 1904 compiled the state of knowledge in a short, yet concise form. The area of application for the "regulations" was geographically restricted to the Prussian empire.

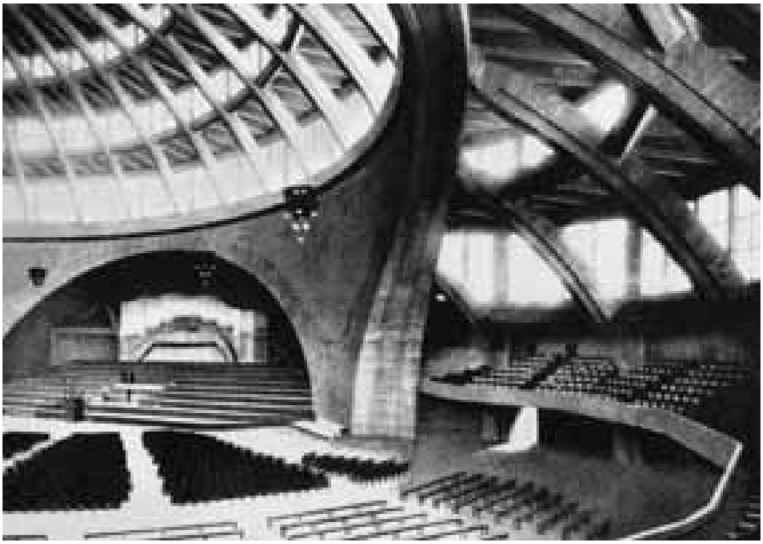

Figure 2. "Jahrhunderthalle" in Breslau (built from 1910 to 1913).

Hence, the knowledge at the beginning of the 20th century was limited to the bending and shear behaviour of reinforced concrete structures. This was extended to the load bearing capacity of reinforced and unreinforced concrete structures in further scientific works. The main research focused on continuous beams made of reinforced concrete, slab structures including

Figure 3. Experimental research of a continuous beam with crack pattern.

the problem of perforation, as well as the load bearing capacity of thrust members with and without buckling.

A further crucial point was the advancement of material properties of concrete properties. The result was the "regulations of the German commission for ferro concrete from September 1925", which consisted of four parts. Essential improvements in these regulations included the consideration of so-called "high-quality" concrete, which was defined as having a cube compressive strength (cube with edge length of $30\,[\mathrm{cm}]$) of at least $27.5\,[\mathrm{N\,mm^{-2}}]$ after 28 days, the consideration of steel with improved strength properties compared to the so far common "Handels-Flusseisen" (St 48 with a tensile strength of at least $180\,[\mathrm{N\,mm^{-2}}]$) and the requirement of a minimum proportion of cement for ferro concrete to protect the reinforcement from corrosion. Furthermore, the allowable stress distinguished four representative hazard categories: For example the safety factor ν of the derivation of the allowed concrete compressive strength σ_{all} increased from $\nu = 2.0$ in the category a) for common building structures to $\nu = 5.0$ for railroad solid-web girder bridges in the category d). This proves that there has been great progress in structure shaping that is to say in the description of the impact of load, since 1904. These advances were observable in more filigree structures compared to the earlier years.

About at the same time (1926) a dissertation by Max Meyer on the topic "The reliability of structures and their calculation on the basis of ultimate force instead of allowable stress" was published. This paper pointed out two basic principles of rational design: On the one hand the designs in the respective limit state and on the other hand the acquisition and the reduction of scatter of the different parameters, such as mechanical properties, impact, geometrical dimensions etc. The development of the concrete construction standard was not influenced by such probabilistic demands up to that time. A classification into arithmetical ultimate load and serviceability load states had not been conducted to date.

The new edition of part A of the "regulations" from 1943 distinguished, for the first time, between four quality categories of concrete: Concrete C120, C160, C225 and C300. The numeric character behind the abbreviation "C" refer to the average cube compressive strength W28, which the concrete has to exhibit after 28 days as measured by using cubes with an edge length of $20\,[\mathrm{cm}]$ or $30\,[\mathrm{cm}]$. Likewise, the reinforcing steel was divided into quality categories, namely into four groups of minimum tensile yield points. The minimum tensile yield point ranged between $200\,[\mathrm{N\,mm^{-2}}]$ (group I) and $500\,[\mathrm{N\,mm^{-2}}]$ (normalized or cold-worked reinforcing steel, group IV). With the application of high-quality raw material and consequently improved bearing capacity of the construction material concrete, the guaranteed minimum strength and quality control became important. These first probabilistic approaches to concrete properties were investigated by Rackwitz, among other things.

By applying this knowledge, based on statistical analyses, a higher quality of the construction material concrete could be attained through quality control. This resulted in a complete

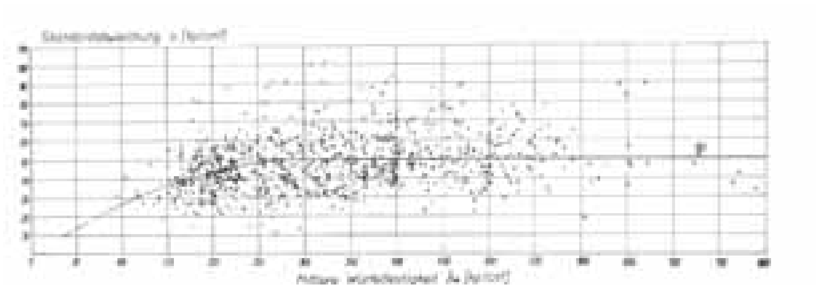

Figure 4. The variation of the compressive strength of concrete cubes (Rackwitz 1971).

revision of the effective design bases. The version of the DIN 1045 from 1972 marked the renunciation of large parts of the design bases that had been authoritative for the design of reinforced concrete structures over the last 40 years, namely from 1932 to 1972. In this context, the introduction of concrete strength classes based on quantile values and their extension to the concrete class B55 should be mentioned. Furthermore, there was a change to a n-free design, which determines the internal forces variable that can be taken up by the cross section under consideration of the arithmetical deformation behaviour of concrete and reinforcing steel, as well as a realistic design for shear force and torsion on the basis of "cracked" framed structure models.

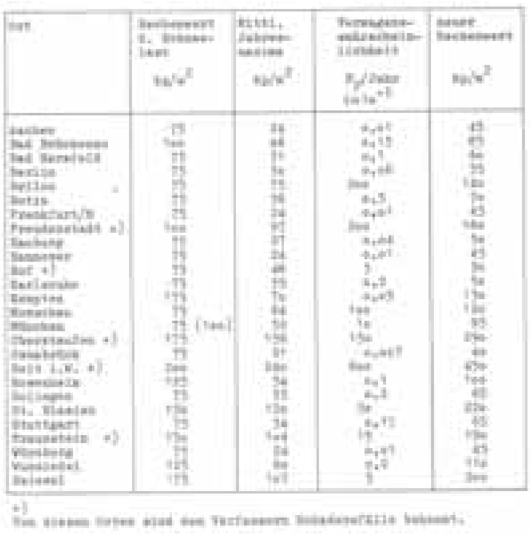

Figure 5. Some aspect of a realistic determination of snow action (Müller and Rackwitz 1973).

2.2 *Determination of Action*

In addition to the properties concerning construction material, the actions on structures are a subject to stochastic modeling. The consideration of the occurring actions is imperative for a reliable prediction of the safety of structures. In the past the magnitudes of the respective actions was defined by legal authorities on the basis of empirical values. This unsatisfying solution was already challenged by Rackwitz in 1973 (Müller and Rackwitz 1973). In that