

Wolfgang Seefelder

Lunar Transfer Orbits utilizing Solar  
Perturbations and Ballistic Capture



Herbert Utz Verlag · Wissenschaft  
München

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Zugleich: Dissertation, München, Techn. Univ., 2002

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ISBN 3-8316-0155-0

Printed in Germany

Herbert Utz Verlag GmbH, München

Tel.: 089/277791-00 – Fax: 089/277791-01

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# 1 Introduction

Preliminary orbit determination is one of the basic challenges of all mission analysis activities for any space mission. It is the process of formulating the fundamental elements and parameters that define a space mission orbit, determining how to get a spacecraft to its destination. This task is, on the one hand, subject to the satisfaction of the inherent objectives, allowing for the achievement of all mission goals. On the other hand, it has a major impact on the design of the spacecraft, as well as on the effort necessary for its operation. In particular the amount of propellant which must be stored in a spacecraft's fuel tanks in order to reach its target, is one of the most crucial conclusions returned from the preliminary orbit determination.

Within this context, the subject of the methods of orbital transfer receives a special attention. As for the problem of a transfer between two celestial bodies, the classical approach suggests at least two definite thrust maneuvers, accounting for the injection from the departure body toward the target body, and for the injection into the final orbit around the target body. It has been the subject of many optimization analyses in the past, to investigate the problem whether two or more maneuvers guarantee a higher efficiency. But what remained constant during all of the investigations concerning this problem, was the requirement of a definite thrust maneuver to leave the gravitational attraction of the departure body and a definite thrust maneuver to achieve a capture at the target body. In this respect, the ideal trajectory is considered to be the one which results from the gravitational attraction only between the spacecraft and a central body. The decision on which celestial body serves as the attracting central body, defining the spacecraft's motion, is driven by the selection of the predominating gravitational environment. Perturbations of any kind are, most commonly, accounted as influences which cause deviations from the nominal two-body trajectory, and, therefore, raise the necessity of correction maneuvers. These may contribute to an increased propellant budget.

In 1991, the Japanese satellite, HITEN, developed from The Institute of Space and Astronautical Science, ISAS, entered an orbit around the Moon without a deterministic thrust maneuver, KAWAGUCHI et al. (1995). On board this spacecraft there was a cosmic dust counter experiment, the Munich Dust Counter (MDC), developed at the Division of Astronautics, Technische Universität München, Germany, IGENBERGS et al. (1991). This was the world's first demonstration of a new class of transfer trajectories which represent a low energy alternative to the classical transfer orbit methods like the Hohmann, or bi-elliptic transfer.

It was originally invented by BELBRUNO (1987) and was first applied to a study of an electric spacecraft mission, LGAS (Lunar Get Away Special), conducted at the Jet Propulsion Laboratory, JPL in 1986-87. The goal of this mission analysis was to investigate the trajectory profile for a small electric propulsion space vehicle, ejected from a canister on

board the space shuttle. The satellite was supposed to slowly spiral away from the Earth and to eventually move into a polar orbit around the Moon. The thrust level, provided by the onboard electric propulsion system, was too small to achieve a capture in terms of a classical final orbit insertion maneuver using the spacecraft engine. This problem has driven the motivation for the discovery of lunar ballistic capture orbits. The fundamental difference between these trajectories and the classical methods of orbital transfer consists of the transformation of the hyperbolic arrival orbit into an elliptic arrival trajectory. For that purpose it is extensively made use of perturbations, in particular of those which are imposed through the gravitational attraction of third-bodies. Hence, for this type of orbits, perturbations do no longer just cause deviations from an ideal trajectory which must be corrected spending additional fuel, but they contribute actively to lower the velocity requirement of spacecraft missions even to values beneath limits which were estimated to be absolute minima in many analytical considerations. Furthermore, the requirement of a definite thrust maneuver to achieve a capture at the target body is eliminated. On the other hand, this type of transfer trajectories strongly affect the orbital conditions at departure and arrival, and it is not yet demonstrated whether each of these parameters can be selected freely.

For this transfer method, being applicable to general mission analyses, it is necessary to prove that solutions can be found which satisfy both, given conditions at the departure point, as well as given conditions at the target body arrival. In this respect, it is the goal of this work to develop strategies and algorithms to calculate ballistic capture transfer trajectories between the Earth and the Moon for a selected Earth departure orbit and selected lunar orbit parameters. These transfer orbits make use of effects which occur in the gravitational interaction of the four-body problem, leading to a chaotic behavior of the spacecraft in the so-called Weak Stability Boundary regions, BELBRUNO (1999). The spacecraft dynamics, connected to the Weak Stability Boundary regions is not yet fully understood. Therefore, it is furthermore the task of the present study, to derive conclusions on the effects of solar perturbations on the utilized lunar ballistic capture orbits. In order to achieve these goals, this work is built of the following structure.

The theoretical background which is necessary to compute and optimize lunar ballistic capture orbits is presented in Chapter 2. It is indicated which time and coordinate systems are used and how transformations can be done between different systems. The basic differential equations of motion which allow for a mathematical modeling of satellite trajectories are derived and some fundamental solutions are presented. Techniques, how to account for perturbations and how to numerically integrate the equations of motion are shown. Finally, the optimization method which has been utilized within this work is briefly described.

Chapter 3 presents an overview of standard transfer methods which are commonly applied to modern space missions. This includes the classical two and three-impulse transfer orbits, the Hohmann and bi-elliptic transfers, and gives an estimate of the global minimum energy Earth-to-Moon transfer trajectory using the circular restricted three-body problem. The state of the art of lunar ballistic capture orbits is shown and an explanation to the term 'Weak Stability Boundary' is given. Within this chapter, the transfer orbit conditions of the selected Earth-to-Moon mission scenario are outlined, indicating the aspects which have not been addressed so far in the context of lunar ballistic capture orbits.



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An analytical estimation of the effect of the gravity potential of the Sun on lunar transfer orbits is presented in Chapter 4. Using a linearization of the equation of motion, regions are determined in which a space vehicle can benefit from solar perturbations in order to reduce its velocity requirement. In this way, regions, defined by the geometrical constellation of the position of the Sun with respect to the spacecraft orbit, can be obtained in which Weak Stability Boundary transfers are possible. They can be deployed to isolate solution candidates during numerical simulation.

Before specific examples of lunar transfer orbits utilizing solar perturbations and ballistic capture will be calculated, evidence of the accuracy of the developed methods and techniques is provided in Chapter 5 by processing real trajectory data. In this way, the used models and deduced analytical relationships, and therefore, the results of this work are validated. For that purpose, operational flight data from the Japanese HITEN and NOZOMI space missions is used. Both satellites have utilized effects connected to the Weak Stability Boundary methodology. By a partial reproduction of these space mission orbits, the numerical force model of the developed software packages, as well as the analytical propositions which are made in order to account for the general feasibility of WSB transfer orbits, can be verified. On board both spacecraft, the Division of Astronautics has placed and operated two cosmic dust counter experiments.

In Chapter 6, the strategies and algorithms which have been used to calculate and optimize Sun perturbed lunar transfers are presented. The results of this process are shown as solutions of the patched conics approximation, as well as the numerical integration. The patched conics approximation allows to determine the order of magnitude of lunar transfer parameters and it is used here as an initial guess for the final optimization using numerical integration.

A systematic search for lunar ballistic capture orbits is presented in Chapter 7. The strategies and algorithms which have been employed in this context are shown. They make use of the results obtained from the analytical estimation in Chapter 4 and the numerical optimization in Chapter 6. Some selected solutions are presented and compared to the results of the Sun perturbed lunar transfers. Within this chapter, the nature of ballistic capture orbits and the behavior of a satellite connected to the Weak Stability Boundary regions are indicated.



## 2 Fundamentals of Astrodynamics

'Astrodynamics is the study of motion of man made objects in space, subject to both natural and artificially induced forces', GRIFFIN & FRENCH (1991) This definition actually combines features of celestial mechanics with orbital and attitude dynamics. Often, these are specified as separated parts of astrodynamics: celestial mechanics examines the dynamic motion of celestial objects, orbital mechanics studies the motion of all orbiting bodies, and attitude dynamics deals with the orientation of an object, VALLADO (1997). As for the problem of a spacecraft transfer from one celestial body to another, both celestial mechanics and orbital mechanics must be applied. This chapter shall provide the basic relationships of astrodynamics which are used to design a transfer from the Earth to the Moon, using the Weak Stability Boundary methodology. As a basis for this chapter, the *Fundamentals of Astrodynamics and Applications* VALLADO (1997) as well as the *Fundamentals of Astrodynamics* BATE et al. (1971) have been used.

### 2.1 Time

Time is the fundamental dimension in almost every branch of science. According to NEWCOMB (1960), 'the main purpose of time is to define with precision the moment of a phenomenon.' This moment is referred to as the epoch of the event. Thus, the epoch designates a particular instant described as a date. The determination of an epoch is based on measuring or counting precisely time intervals. In astrodynamics, time is particularly critical, because objects are far and move quickly. To have a practical time system, it is necessary to have a repeatable time interval based upon some measurable physical phenomena and also to have a fundamental epoch from which to count intervals. The commonly accepted fundamental epoch is the beginning of Christian era, although others exist. Finding a precise repeatable time interval is more problematic. Four time systems now provide timekeeping for scientific, engineering, and general purposes. These are:

- Sidereal Time
- Solar and Universal Time
- Dynamical Time
- Atomic Time

Sidereal time and universal time are based upon the Earth's rotation and are related through mathematical relationships. Dynamical time and atomic time are truly independent from the other forms. They are for very precise timekeeping.