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**Cyclic plasticity modeling  
and multiaxial fatigue assessment for an  
austenitic steel**



Herbert Utz Verlag · München

## Werkstoffwissenschaften



Zugl.: Diss., Wuppertal, Univ., 2014

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ISBN 978-3-8316-4484-1

Printed in EU  
Herbert Utz Verlag GmbH, München  
089-277791-00 · [www.utzverlag.de](http://www.utzverlag.de)

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# Abstract

Many key components of engineering structures are exposed to repeated and combined service load sequences, resulting in fatigue failure. Notches and geometrical irregularities contained in those components cause stress concentrations and accelerate fatigue failure processes. Reliable methods of evaluation are necessary, which can account for notch effects even under uniaxial fatigue and complex variable loading amplitudes. An important requirement to support these methods is the local stress and strain distributions. A suitable constitutive model is necessary for an accurate prediction of the fatigue life.

Experimental investigations show that SS304 stainless steel exhibits an initial cyclic hardening followed by a softening period, without cyclic stabilization. For higher strain amplitude above 1%, secondary hardening can be observed. The material displays strain range dependent cyclic hardening as well as significant non-proportional additional hardening. Systematic fatigue experiments are performed and analyzed including uniaxial and multiaxial fatigue tests of smooth specimens and notched specimens. Due to the mean stress relaxation, the loading ratio effect on the fatigue damage is not significant in the low cycle fatigue life.

Fatemi-Socie, Brown-Miller, Findley, Cruse-Meyer and Smith-Watson-Topper fatigue life models are investigated in the present work to assess SS304 fatigue life. Without considering the additional hardening, the pure strain based models give a non-conservative prediction. Oppositely, the stress based model shows a too conservative prediction for non-proportional loading cases, especially in the low cycle region. The stress and strain mixed model and energy based model can give better results. Both additional hardening and non-proportional effects should be considered in the fatigue life assessment. The Fatemi-Socie fatigue damage parameter is modified through introducing two weight constants, one is for the normal stress amplitude and the other is for the loading path dependent non-proportional parameter. The mean stress and the stress amplitude normal to the critical plane are treated as two independent variables instead of the maximum stress. By comparing experimental data with predictions from all selected damage parameters, the modified model leads to a more satisfactory result.

Several well-known kinematic hardening models and combined hardening models are presented and investigated. Compared with other Armstrong-Frederick type models, Karim-Ohno kinematic hardening model has only one supplementary parameter for better cyclic stress relaxation/ratchetting description purpose. The cyclic hardening of SS304 is mainly induced by the increase of the plastic modulus, which cannot be described by the combined hardening model. Therefore, the present work extends the Karim-Ohno kinematic hardening plasticity model for more complex loading configurations. A new equation is proposed to describe evolution of the kinematic hardening. Especially the ability for processing non-Masing behavior and nonproportional additional hardening has been added. In addition, the evolution equation for so-called strain-memory-effect is implemented, in order to take into account the memory-effect observed in the experiment. This new model is implemented into the commercial FEM code ABAQUS. An implicit

integration algorithm and an expression of consistent tangent modulus are established for the model. It is shown that the introduced model is quite successful in accounting for simulations of monotonic loading, non-Masing behavior, mean stress relaxation, cyclic hardening and non-proportional hardening.

The stress and strain states around the notch root are generally multi-axial and vary with distance even under uni-axial loading condition. It is found that different constitutive models will lead to different fatigue prediction results. And the differences depend on the examined fatigue damage parameter. In the present work, the modified Karim-Ohno model is used to describe the cyclic material behavior of notched specimens. And the modified Fatemi-Socie fatigue damage parameter is adopted to take the multiaxial stress state into account. A nonlocal fatigue criterion based on the average fatigue damage over a specific distance from the notch root is presented and calculated to predict the fatigue lifetime. This distance is treated as a material parameter and can be determined with help of FEM computations illustrated in the present work. Good agreement between the prediction and the experiments has been observed in using the proposed method. This method possesses the potential to unify the fatigue life prediction for both smooth and notched specimens.

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# Zusammenfassung

Viele der wichtigsten Komponenten in Ingenieur Anwendungen werden wiederholten und kombinierten Lastsequenzen ausgesetzt, die in Ermüdungsversagen enden. Kerben und geometrische Irregularitäten, die in diesen Komponenten beinhaltet sind, lösen Spannungskonzentrationen aus und beschleunigen Ermüdungsversagen. Zuverlässige Methoden für Kerbeffekte unter uniaxialer Ermüdung und unter komplexen variablen Belastungsamplituden sind nötig. Eine wichtige Voraussetzung, um besagte Methoden zu unterstützen, ist die Klarheit über die lokalen Spannungs- und Dehnungsverteilungen. Ein passendes konstitutives Modell für eine akkurate Beschreibung der Ermüdungslebensdauer ist nötig.

Experimentelle Beobachtungen bestätigen, dass Chromnickelstahl SS304 eine initiale zyklische Verfestigung gefolgt von einer zyklischen Entfestigungsperiode ohne zyklische Stabilisierung aufweist. Für höhere Dehnungsamplituden über 1% kann sekundäres Verfestigungsverhalten beobachtet werden. Das Material zeigt demnach ein zyklisch transientes Verformungsverhalten, das vom Dehnungsbereich abhängt, und außerdem ein signifikantes nichtproportionales Verfestigungsverhalten. Systematische Ermüdungsexperimente, die uniaxiale sowie multiaxiale Ermüdungstests von ungekerbten und gekerbten Proben beinhalteten, wurden durchgeführt und analysiert. Infolge der Mittelspannungsrelaxation ist der Effekt des Spannungsverhältnisses auf die Ermüdungsdauer im Kurzzeitverhalten nicht signifikant.

Die Ermüdungslebensdauermodelle von Fatemi-Socie, Brown-Miller, Findley, Cruse-Meyer und Smith-Watson-Topper wurden zur Bewertung der Schwingfestigkeit des Chromnickelstahls SS304 untersucht. Reine dehnungsbasierte Modelle können die Zusatzverfestigung infolge nicht-proportionaler dehnungskontrollierter Beanspruchung nicht erfassen. Im Gegensatz dazu zeigen spannungsbasierte Modelle eine sehr konservativere Vorhersage für nicht-proportionale Belastungsfälle, insbesondere im Kurzzeitfestigkeitsbereich. Mit Schädigungsparametern, die sowohl die Spannung als auch die Dehnung beinhalten, sowie energiebasierten Modellen können bessere Ergebnisse erzielt werden. Nichtproportionale Belastungen resultieren nicht nur in zusätzlicher Verfestigung, sondern auch in einer frühen Initiierung von Ermüdungsrissen. Demnach sollte die nicht-proportionale Zusatzverfestigung berücksichtigt werden. Der Fatemi-Socie-Ermüdungsschädigungsparameter wurde hier durch die Einführung zweier Gewichtskonstanten, eine zur Gewichtung der Normalspannungsamplitude und die andere zur Gewichtung des pfadabhängigen Nichtproportionalitätsparameters, modifiziert. Mittelspannung und Spannungsamplitude normal zur kritischen Ebene werden als zwei getrennte Teile im Modell behandelt, anstelle der Maximalspannung, die im konventionellen Modell verwendet wird. Beim Vergleich zwischen experimentellen Daten und Vorhersagen, die durch die Auswahl von allen Ermüdungsschädigungsparameters berechnet wurden, führten die modifizierten Modelle zu zufriedenstellenden Ergebnissen.

In der Arbeit werden einige bekannte kinematische und gemischt isotrop-kinematische Verfestigungsmodelle analysiert. Verglichen mit anderen Modellen vom

Armstrong/Frederick-Typ, sorgt ein zusätzlicher Parameter im Karim-Ohno-modell für eine bessere Modellierung des zyklischen Kriechens und der zyklischen Mittelspannungsrelaxation. Die zyklische Verfestigung von SS304 wird hauptsächlich durch die Zunahme des plastischen Tangentenmoduls induziert, die das gemischt isotrop-kinematische Verfestigungsmodell nicht repräsentieren kann. Deshalb erweitert diese Arbeit das kinematische Verfestigungsplastizitätsmodell nach Karim-Ohno für komplexere Belastungskonfigurationen. Eine neue Evolutionsgleichung für die Beschreibung der kinematischen Verfestigungsparameter wird vorgeschlagen. Insbesondere wurde gegenüber anderen Ansätzen zusätzlich die Modellierung von Nicht-Masing-Verhalten auch bei nichtproportionaler Verfestigung ermöglicht. Weiter wird die Entwicklungsgleichung für den Dehnungs-Gedächtnis-Effekt implementiert, um im Experiment beobachtete Gedächtniseffekte zu berücksichtigen. Dieses Modell ist in den kommerziellen FEM-Code ABAQUS implementiert worden. Es wird gezeigt, dass das modifizierte Modell beim Nachweis für die Simulation von monotonen Belastungen, Nicht-Masing-Verhalten, Mittelspannungsrelaxation, zyklischer Verfestigung sowie nicht-proportionaler Verfestigung ziemlich erfolgreich ist.

Die Spannungs- und Dehnungszustände im Bereich des Kerbgrunds sind normalerweise multiaxial und variieren mit der Distanz sogar unter uniaxialen Belastungsbedingungen. Es wird festgestellt, dass verschiedene konstitutive Modelle zu unterschiedlichen Ermüdungsvorhersageergebnissen führen. Und die Unterschiede hängen von den angewandten Ermüdungsschädigungsparametern ab. In der vorliegenden Arbeit wird das modifizierte Karim-Ohno-Modell für die Beschreibung des zyklischen Materialverhaltens gekerbter Proben benutzt. Bei der Vorhersage der Schwingfestigkeit wird das modifizierte Fatemi-Socie-Modell verwendet, um einen multiaxialen Spannungszustand zu berücksichtigen. Ein nichtlokales Ermüdungskriterium basierend auf dem mittleren Ermüdungsschaden in einem spezifischen Bereich des Kerbgrunds wird vorgestellt und berechnet, um die Ermüdungslebensdauer vorherzusagen. Diese Entfernung wird als Materialparameter behandelt. Eine Methode zur Bestimmung der Entfernung ist auch in dieser Arbeit gezeigt. Es konnten gute Übereinstimmungen zwischen den Vorhersagen mit den vorgestellten Verfahren und den experimentellen Daten gefunden werden. Dieses Modell besitzt das Potential, die Ermüdungslebensdauervorhersage für ungekerbte und gekerbte Proben zu vereinheitlichen.



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# 1 Introduction

## 1.1 Background

Development of new products such as nuclear power plants, automobiles and aircrafts, raises high demands for the integrity and reliability of structures. With regards to integrity, one of the important requirements is the durability, which deals with the capability of structures subjected to loads in service without failure occurring. Loads, material properties, geometry, manufacturing processes and service conditions can all affect the durability of the structure. The majority of durability failures is caused by fatigue. Fatigue takes place when a structure subjected to repeated or cyclic loading, which results in a progressive localized permanent structural change. After a sufficient number of loading cycles cracks may nucleate or even complete fracture may cause structural failure [1]. One famous example occurred in 1954, two Comet aircrafts crashed after 1286 and 903 flights due to metal fatigue failure of the fuselage structure [2]. The fatigue forms of metallic material include high cycle fatigue(HCF) and low cycle fatigue(LCF) caused by fluctuating stresses and strains, creep fatigue induced by repeated loads at high temperature, thermal mechanical fatigue due to combination of thermal and mechanical loads, fretting fatigue caused by repeated sliding between two contact surfaces and corrosion fatigue due to the joint interaction of corrosion and cyclic loads. High cycle fatigue and low cycle fatigue are the most common form of fatigue. HCF is characterized by the low stress amplitude with elastic strains at high frequency, while LCF is described by the high stress amplitude with plastic strains. For example, LCF in aircraft engines is generally coupled with flight missions in a combination of take-off, cruise and landing, whereas HCF is generally based on cycles with high frequent vibration or rotation.

Until now most of fatigue criteria are based on strain/stress. The fundamental idea of these models is that fatigue life can be determined by examining effects of fluctuating strain/stress or combination of them, such as energy or critical plane. At present, no single method is appropriate for all multi-axial fatigue situations, especially under non-proportional loading. Detailed knowledge of stain/stress distributions of the structural component is necessary for the fatigue assessment. But the conventional constitutive models do not provide an accurate description of the complex material mechanical behaviors. Therefore, many modified models were proposed by incorporating more material mechanics description. The validity of their applications might be limited to several loading conditions [3]. Notches in mechanical parts cause stress concentrations and accelerate material fatigue failure. Reliable methods to account for notch effects even under uniaxial fatigue and complex variable loading amplitude are necessary. In conventional fatigue models, multi-axial stress/strain state and their gradient effects near the notch root were not considered properly.

## 1.2 Development of cyclic plasticity

Generally speaking, the fatigue life until failure consists of three periods: crack nucleation, crack propagation and final fracture [4]. Fatigue crack nucleation is the result of cyclic slips. It implies cyclic plastic deformation, or in other words, dislocation activities. On the surface of a metal body, there will be intrusions and extrusions induced by slip band movement subjected to cyclic loading. The intrusions form stress concentrations, which can be the locations of crack nucleation. Fatigue failure is a process of cyclic stress/strain evolutions and redistributions in the critical stressed volume [5].

Although most engineering structures are designed to subject nominal elastic loads. Stress concentration caused by notches often leads to plastic deformation near the notch root. This plastic zone is surrounded by elastic material, which forms boundary constraints. Deformation at the notch root is considered as strain controlled. Due to the concentration effect, the local material in the vicinity of the notch root yields firstly and influences the surrounded material. To achieve a realistic assessment of the low cycle fatigue life, knowledge of the cyclic stress/strain evolution and redistribution is essential. For the homogeneous material subjected to uniaxial cyclic loading, Neuber's rule[6], as well as Molski and Glinka equivalent strain energy density method (ESED) [7] are widely used in determination of the notch root strain/stress. In these approximation methods, elastic analysis results of strain and stress in the concentration region are used to predict the elastic-plastic strain and stress. Studies of Moftakhar et al. [8] showed that Neuber's rule overestimates the notch-tip elastic plastic strains and stresses and the ESED method underestimates the notch-tip inelastic strains and stresses. For multiaxial stress/strain states, approximate solutions have been proposed by Dowling [9] and Walker [10].

With a proper constitutive model, finite element method (FEM) can provide more reliable solutions for the stress/strain calculation in a notch under complex loading conditions [11]. In order to improve the accuracy of fatigue life prediction, the plastic material behavior must be taken into account. Thus, a reliable and accurate prediction of the nonlinear stress-strain relation of materials subjected to cyclic loading conditions is required.

Cyclic plasticity is handled by the external cyclic loading, which results in a nonlinear stress-strain relationship. Under cyclic loading the stress-strain relation can be very different from that in a monotonic tension or compression. Bauschinger effect, cyclic hardening or softening, non-proportional additional hardening, and mean stress relaxation or ratchetting are known phenomena during cyclic deformation [12–19]. Early studies of cyclic plasticity concentrated mainly on the monotonic and uniaxial loading conditions. But in the past four decades, efforts have been directed toward cyclic multi-axial plasticity for both proportional and nonproportional loading.

In the past years, many constitutive models for the description of cyclic inelasticity have been published in literature. The first work was attempted by Prager [20], in which a linear kinematic hardening rule was proposed. This model makes it possible to describe the Bauschinger effect, but fails to predict other phenomena. Afterwards, modifications proposed by Besseling [21] and Mroz [22] was introduced by considering a multi-surface concept. Following this concept, Krieg [23], Dafalias and Popov [24], McDowell [25], Ohno and Kachi [26], Moosbrugger and McDowell [27] made a lot of improvement to represent more cyclic phenomena. But the Mroz's type models has mathematical shortcomings in dealing with non-proportional loading [28, 29].

In order to produce a non-linear evolution law, a dynamic recovery term was introduced into the linear kinematic hardening by Armstrong and Frederick [30]. This modification was found to be important and a lot of kinematic hardening rules were proposed based on this idea. The Armstrong and Frederick model was built based on a physical mechanism of strain hardening behavior and can describe the hysteresis loops well. The ratchetting behavior is represented by the evolution of the backstress, but this model gives very significant ratchetting in general. Later, Chaboche [31] made a significant modification to the Armstrong and Frederick model, and decomposed the back stress into several parts. This modification enables this model to express the cyclic hardening in different length scales, ranging from cell walls to grain boundaries or clusters of microtexture [32]. Then Chaboche [31] improved the over-estimate in ratchetting by reducing the effect of the recovery term by setting a threshold, which allows dynamic recovery only when a backstress reaches a critical value. This model predicts less ratchetting than the Armstrong and Frederick model. Ohno and Wang [33] used the concept of critical state to limit the effect of the recovery term for each decomposed part of the backstress. The modification gives no or little ratchetting under uniaxial cyclic loading. The critical state enables the integration of the backstress more accurately and efficiently [34]. After that, there were many proposed evolution equations for better ratchetting prediction in the framework of nonlinear kinematic hardening rules. Most of them are based on the decomposition of the backstress. Bari and Hassan [35] designed a new evolution rule by superposing the Chaboche model with the Burlet-Caletaud model [36], which provides an improved simulation to the biaxial ratchetting. Jiang and Sehitoglu [28] developed a new model with a controlling exponent, which enables it to model decaying ratchetting. Chen and Jiao [37] developed a kinematic hardening rule by superposing the Ohno-Wang model upon the Burlet-Caletaud model, which provides a reasonable simulation for non-proportional multi-axial ratchetting. By introducing a softening index surface, Yagochi and Takahashi [38] successfully extended the Ohno-Wang model to express the ratchetting of the material with cyclic softening behavior. With regard to steady-state ratchetting description, Abdel-Karim and Ohno [39] proposed a combination of the Ohno-Wang model and Armstrong-Frederick model. This model can represent well the steady-state ratchetting both in uni-axial and multi-axial loadings. For long-term ratchetting under a relatively large number of stress cycling, accurate modelling of ratchetting is still difficult. A detailed review of various non-linear kinematic hardening models can be found in Chaboche's paper [3].

For description of cyclic hardening/softening behavior, isotropic hardening is often carried out by using the accumulated plastic strain to describe the size change of the yield surface. Chaboche [40] proposed a combined hardening model to describe this behavior by adding a hardening component to the evolution of the yield surface. This model can simulate monotonic hardening and the Bauschinger effect. Jiang and Sehitoglu [28] represented this transient behavior by varying the coefficients in the kinematic hardening relation as a function of the accumulated plastic strain. The non-Masing behavior was considered through the yield stress as a function of a stress memory surface size. But it cannot model the non-proportional hardening. To overcome this limitation, Döring [41] related the critical state parameter in the Jiang model to the loading history to consider the cyclic hardening and non-proportional hardening. Through the changing of the critical state parameter, the transient behavior can be modeled over a wide range. Tanaka's [42] non-proportionality factor was introduced to indicate the non-proportionality. The

memory surface defined in the plastic strain space proposed by Chaboche [43] was applied to track the loading amplitude. This model can give a better description of most cyclic plasticity phenomena of low alloy steels, e.g. S460N, especially for the ratchetting with high number of cycles. The shortcomings of this model are that it can not present a steady-state ratchetting and the parameter identification is too complex due to the tracking of each critical state parameter variation at different strain amplitudes. Based on the coupled nonlinear kinematic hardening model proposed by Bari and Hassan [35], Krishna and Hassan [44] proposed a modified Chaboche model. Both the plastic strain memory surface of Chaboche and non-proportionality defined by Tanaka were incorporated. For most plastic phenomena, this model simulated much better, it fails to simulate the hysteresis loop shape and size of the uniaxial ratchetting.

Recently, Abdel-Karim [45] extended the Ohno-Wang model to incorporate isotropic hardening. The isotropic hardening was associated with the kinematic hardening through changing the critical surface parameter in the initial Ohno-Wang model. But the plastic strain range dependent behavior was not incorporated. Kang [46] made a lot of works in the simulation of cyclic plasticity. Kang and Ohno [47] assumed that the isotropic hardening can be decomposed into several parts and the evolution of each part is in the critical state. This extended model can simulated the strain range dependent cyclic hardening behavior. Kang and Yang [19] modified the dynamic recovery term in Armstrong and Frederick and gave a new model, in which the memorization of the plastic strain and the effect of non-proportionality on isotropic hardening were included. By employing the Abdel-Karim-Ohno model into the framework of viscoplastic, time-dependent ratchetting was described in Kang's work [48]. Then isotropic hardening was also incorporated by Kang and Kan [49] based on the Abdel-Karim-Ohno model. Ahmazdazeh [50] developed the Armstrong and Frederick model with Bower's [51] modification by involving new ratchetting rate coefficients. This modified hardening rule successfully represented constant ratchetting strain rate and the followed decaying strain rate. Khutia et al [52] modified the Ohno-Wang model by incorporating a new fading memory stress function. The fading memory stress is used to model cyclic hardening/softening and non Masing behavior. This model was proved to be better than the Ohno-Wang model in ratchetting prediction. But it has difficulties in description of hysteresis loop related to the non-Masing behavior, and non-proportional hardening is not included. Recently, Chuang and Park [53] attempted to extend Chaboche model for anisotropic yield functions. They found that it will ultimately accounts for the isotropic hardening of an anisotropic yield function, when fully imposed the consistency condition.

Until now, most models are developed phenomenologically based on the macroscopic experimental results. Owing to the complexity of phenomena in cyclic deformation, there is no model which can describe all the phenomena during cyclic deformation well. For general application of a stable material, Armstrong and Frederick, Chaboche and Ohno-Wang models are capable to capture the essential phenomena. For more complex behavior, such as cyclic hardening, non-proportional hardening and non-Masing behavior, additional evaluation equations have to be incorporated. But more equations will bring difficulties in numerical solving. The implementation of an advanced plasticity theory to capture the mechanical behavior under general loading is an additional challenge. Cyclic plastic deformation is an essential component of the fatigue damage process. Therefore, understanding of multi-axial cyclic plastic deformation is often necessary, particularly in situations when significant plasticity exists such as at notches and in low cycle fatigue.

## 1.3 Multiaxial fatigue

Multi-axial fatigue refers to the material failure process under multiaxial cyclic loading conditions. There are at least two or three stresses or strains independently applied to a body under multiaxial cyclic loading conditions [54]. They change with time independently, and their ratio can be proportional or non-proportional. Compared with uniaxial fatigue, it's more complex for multiaxial fatigue in mechanics analysis, experimental research and failure mechanisms. Under multi-axial fatigue, the theory of fatigue crack nucleation and propagation direction, fatigue life and fatigue damage accumulation needs more improvement. Especially under non-proportional loading condition, the rotation of the principal stress and strain cause plastic deformations along several different slip systems. Compared with proportional loading condition, more slip systems will be activated and result in more damage under non-proportional loading [55]. Both mean value and out of phase of each loading component can cause non-proportional loading case [56].

According to the major physical quantity the theories of multiaxial fatigue can be classified as follows: stress based, strain based, energy based, and fracture mechanics based models. The stress based approach is limited to the high cycle fatigue regime where plastic stains may be negligible. The strain based approaches can cover both low and high cycle fatigue regimes. The early work of Morrow on cyclic plasticity and energy serves as basis for many of the multiaxial energy models. Energy is a scalar quantity and does not address the orientation of crack nucleation and propagation. The linear elastic fracture mechanics are used to predict fatigue growth life and fracture. The crack growing process is described based on fracture mechanics concept.

Experimental observations indicate that, depending on the material and loading conditions, crack propagation are on either maximum shear planes or maximum tensile stress planes, or an empirical maximum plane. Fatigue models relating fatigue damage to stresses and/or strains on these planes are called critical plane model. These models can predict not only the fatigue life, but also the orientation of the crack or failure plane. Different damage parameters using stress, strain, or energy quantities have been used to evaluate damage on the critical plane.

Findley [57] identified a critical plane based on a combination of shear stress amplitude and maximum normal stress. Dang Van [58] proposed a macro-micro-approach, which works well in high cycle multiaxial fatigue. Liu and Mahadevan [59] showed a characteristic plane approach, which performs well under different failure mechanisms of metals. These stress-based damage parameters are suitable for high cycle fatigue regime where the plastic deformation is negligible.

Similar to Findley's work, Brown and Miller [60] proposed that both the shear strain amplitude and normal strain on the maximum shear strain plane should be considered. Fatemi and Socie [61] modified this parameter by replacing the normal strain in the maximum shear strain plane with the maximum normal stress. Smith et al. [62] (SWT) proposed a parameter which includes the shear strain range and the maximum principle stress. Socie [63] introduced the critical plane concept into the SWT parameter. Li et al. [64] introduced the maximum normal stress on the maximum shear strain range plane to the Brown and Miller model. Based on equivalent strain method, Shang and Wang [65] proposed a parameter to consider the effect of loading path. In these models, the contained normal stress term can be taken into account the mean or hydrostatic stress

effects and give a better prediction for materials with non-proportional hardening. The strain energy may be considered as a fatigue damage parameter. Based on this idea, Liu [66] proposed the virtual strain energy in the critical plane. In this model, the different influences of the normal and shear strain energy density to the fatigue damage is not considered. By considering that shear mean stress has little effect on fatigue strength, Glinka et al. [67] proposed an energy parameter, which is a part of total strain energy density. Varvani-Farahani [68] proposed a parameter which adds normal and shear strain energy together and weights by the axial and shear fatigue properties.

Except these methods, a more recent method based on microstructural fracture mechanics will also be evaluated to predict fatigue life. Socie and Furman [69] proposed a short crack method to take account the crack growth and also the initiation. Then Hoshide and Kusuura [70] employed a J-Integral-based formulation. Closure behaviour of short cracks was then considered by Vormwald and Seeger [71]. Döring et al. [72] proposed a short crack model for non-proportional loading. Hertel and Vormwald [73] extended this model for notched components under multiaxial variable amplitude loading, and obtained a successful prediction.

Many overviews on multiaxial fatigue can be found in [74–85]. Due to the general various practical applications, the multiaxial fatigue life prediction remains a challenging problem, and additional research studies are required.

### 1.4 Objective and outline

The objective of this work is to investigate a multiaxial fatigue analysis methodology for performing life prediction based on elastic-plastic computation. The objective of the work is accomplished by performing the following four parts:

- Systematically investigation on the material mechanics behaviors under cyclic loadings.
- Establishment of multiaxial fatigue damage parameters to estimate the fatigue life under proportional and non-proportional loading histories.
- Formulation and implementation of a suitable constitutive model into a commercial FEM code, which should give a more accurate description of the essential cyclic mechanical behaviors.
- Development of the life prediction method for notched components based on the studied fatigue damage parameter and the cyclic plasticity constitutive model.

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